Test 3 Review

General Things

- 100 points possible
- NO CALCULATOR of any kind will be needed or allowed
- covers Chapter 12: Sections 12.2 - 12.5, 12.7 - 12.8
- There are 7 questions (including 1 conceptual question)
- Extra Credit is available and will be due on Monday, March 31, 2008.

Section 12.2

- $\int_a^b A(x)dx = \int_a^b \left[\int_c^d f(x,y)dy\right] dx$ known as an **iterated integral**
  - Work from the inside out.
  - Keep one variable fixed(constant) and integrate with respect to the other variable
- Fubini's Theorem: $\iint_R f(x,y)dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$

Section 12.3

- **Type I** - region lies between graphs of two continuous functions of x.
  - $D = \{(x,y)|a \le x \le b, g_1(x) \le y \le g_2(x)\}$
  - $\int\int_D f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$
- **Type II** - region lies between graphs of two continuous functions of y.
  - $D = \{(x,y)|c \le y \le d, h_1(y) \le x \le h_2(y)\}$
  - $\int\int_D f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$

Section 12.4

- Equations for polar coordinates

$$x = r\cos\theta \qquad y = r\sin\theta \qquad x^2 + y^2 = r^2$$

- Use polar coordinates when dealing with circular regions
- Area of small region is $r\,dr\,d\theta$

- $\displaystyle\iint_R f(x,y)dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta) r\, dr\, d\theta$
- convert from rectangular to polar coordinates
  - replace: $x = r\cos\theta$ and $y = r\sin\theta$
  - change limits of integration
  - replace dA with $r\, drd\theta$
- DO NOT forget the r in the $r\, drd\theta$

<u>Section 12.5</u>

- For a lamina with density $\rho(x,y)$ the total mass is given by $m = \displaystyle\iint_D \rho(x,y)dA$
- Electric charge distributed over a region D is $Q = \displaystyle\iint_D \sigma(x,y)dA$
- Moment of the entire lamina is
  - $M_x = \displaystyle\iint_D y\rho(x,y)dA$ about the x-axis
  - $M_y = \displaystyle\iint_D x\rho(x,y)dA$ about the y-axis.
- Center of mass $(\bar{x}, \bar{y})$ where $\bar{x} = \dfrac{M_y}{m} = \dfrac{1}{m}\displaystyle\iint_D x\rho(x,y)dA$ and $\bar{y} = \dfrac{M_x}{m} = \dfrac{1}{m}\displaystyle\iint_D y\rho(x,y)dA$.
- Moments of Inertia
  - $I_x = \displaystyle\iint_D y^2\rho(x,y)dA$ about the x-axis
  - $I_y = \displaystyle\iint_D x^2\rho(x,y)dA$ about the y-axis
  - $I_0 = \displaystyle\iint (x^2+y^2)\rho(x,y)dA$ about the origin

<u>Section 12.7</u>

- **Type 1** region lies between the graphs of two continuous functions of x and y.
  - $E = \{(x,y,z) | (x,y) \in D, u_1(x,y) \leq z \leq u_2(x,y)\}$
  - $\displaystyle\iiint_E f(x,y,z)\, dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\, dz\right] dA$
- **Type 2** region lies between the graphs of two continuous functions of y and z.
  - $E = \{(x,y,z) | (y,z) \in D, u_1(y,z) \leq x \leq u_2(y,z)\}$

- $\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(y,z)}^{u_2(y,z)} f(x,y,z)\,dx\right] dA$

- **Type 3** region lies between the graphs of two continuous functions of x and z
  - $E = \{(x,y,z)|(x,z) \in D, u_1(x,z) \leq y \leq u_2(x,z)\}$
  - $tint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x,y,z)\,dy\right] dA$

- If $f(x,y,z) = 1$ then the triple integral represents the volume of E: $V(E) = \iiint_E dV$.

- All of the equations in Section 12.5 can be extended to functions of 3 variables.

Section 9.7/12.8

- Cylindrical coordinates are best used for cylindrical regions

- Know how to convert from rectangular to spherical coordinates and vice versa
  - $x = r\cos\theta \quad y = r\sin\theta \quad z = z$
  - $r^2 = x^2 + y^2 \quad \tan\theta\frac{y}{x} \quad z = z$

- $\iiint_E f(x,y,z)\,dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\,dz\,dr\,d\theta$

- Spherical coordinates are best used for spherical regions

- Know how to convert from rectangular to cylindrical coordinates and vice versa
  - $x = \rho\sin\phi\cos\theta \quad y = \rho\sin\phi\sin\theta \quad z = \rho\cos\phi$
  - $\rho^2 = x^2 + y^2 + z^2$

- $\iiint_E f(x,y,z)\,dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\,\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

Practice Problems

1. Evaluate $\iint_D (x+2)dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.
2. Find the volume of the solid enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0$, $y = 1$, $y = x$, and $z = 0$
3. Evaluate $\iint_R \cos(x^2 + y^2)dA$, where R is the region that lies above the x-axis within the circle $x^2 + y^2 = 9$.
4. Find the mass and center of mass of the lamina that occupies the region D where D is bounded by the parabola $x = y^2$ and the line $y = x - 2$ and has the density function $\rho(x, y) = 3$.
5. Evaluate $\iiint_E 6xydV$ where E lies under the plane $z = 1+x+y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.
6. Use cylindrical coordinates to evaluate $\iiint_E e^z dV$, where E is enclosed by the paraboloid $z = 1 + x^2 + y^2$, the cylinder $x^2 + y^2 = 5$, and the xy-plane.
7. Use spherical coordinates to evaluate $\iiint_E zdV$, where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

Answers

1. $\frac{32}{15}$
2. $\frac{5}{6}$
3. $\frac{\pi}{2}\sin 9$
4. $m = \frac{27}{2}, (\bar{x}, \bar{y}) = \left(\frac{8}{5}, \frac{1}{2}\right)$
5. $\frac{65}{28}$
6. $\pi(e^6 - e - 5)$
7. $\frac{15\pi}{16}$