Test 1 Review

General Things:

- 100 points possible
- NO CALCULATOR of any kind will be allowed
- covers Sections 9.1 - 9.6, 10.1 - 10.4
- Extra Credit is available and will be due on Wednesday, September 19, 2007.

Section 9.1

- Distance Formula in 3D: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Section 9.2

- Add two vectors:
  - **geometrically** by using either the Triangle Law or the Parallelogram Law.
  - **algebraically** by adding the corresponding components of the vectors.
- Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ the the vector between the two points is $\mathbf{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$
- The length of a vector is given by $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.
- A unit vector, $\mathbf{u}$, is a vector whose length is 1. It is found using $\dfrac{\mathbf{a}}{|\mathbf{a}|}$.
- The resultant force is the sum of all the forces acting on an object.

Section 9.3

- The dot product of two nonzero vectors is given by
  - $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$
  - $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$
- Uses of dot product
  - determine the work done moving an object given the force and displacement vectors, $W = |F||D| \cos\theta$
  - find the angle between two vectors
  - the scalar/vector projection of one onto another
  - determine if two vectors are orthogonal, $\mathbf{a} \cdot \mathbf{b} = 0$

Section 9.4

- The cross product of two vectors is given by
  - $\mathbf{a} \times \mathbf{b} = (|\mathbf{a}||\mathbf{b}| \sin\theta)\, \mathbf{n}$

  - $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

- Uses of cross product
  - determine the torque if the force and position vectors are known
  - create a vector orthogonal to two given vectors
  - determine if two vectors are parallel
  - find the area of a parallelogram determined by two vectors
- The volume of a parallelogram determined by **a**, **b**, and **c** is the magnitude of their scalar triple product: $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$

Section 9.5

- To find a vector perpendicular to a plane
  - If equation of plane is known, it can be written as $ax + by + cz + d = 0$. The normal vector perpendicular to the plane is $\langle a, b, c \rangle$
  - If equation is not known, use points on the plane to find two non-parallel vectors which lie in the plane. The cross product of these vectors is a vector perpendicular to the plane.
- vector equation for a line: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$
- parametric equations for a line: $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$
- symmetric equations: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$
- Two vectors are parallel if and only if one is a scalar multiple of the other; also, if and only if their cross product is 0.
- Two vectors are perpendicular if and only if their dot product is 0.
- Two planes are parallel if and only if their normal vectors are parallel.

Section 9.6

- The domain of a function of two variables is the set of points (x, y) that will give $f(x, y)$ a unique real number.

Section 10.1

- A vector function is a function whose domain is a set of real numbers and whose range is a set of vectors. To find the derivative or integral, we can differentiate or integrate each component of the vector function.

Section 10.2

- A curve $\mathbf{r}(t)$ is smooth if $\mathbf{r}'(t)$ is continuous $\mathbf{r}'(t) \neq 0$

- The tangent vector to a smooth curve is the vector $\mathbf{r}'(t)$. A unit tangent vector is given by $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$
- A unit normal vector is given by $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$

Section 10.3

- The length of a space curve is given by $L = \displaystyle\int_a^b \left|\mathbf{r}'(t)\right| dt$
- The curvature of a curve at a given point is a measure of how quickly the curve changes direction at that point. It is calculated by
  - $\kappa(t) = \left|\dfrac{\mathbf{T}'(t)}{\mathbf{r}'(t)}\right|$ $\qquad \kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$ $\qquad \kappa(x) = \dfrac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}$

Section 10.4

- If $\mathbf{r}(t)$ is the position vector of the particle on the space curve, the velocity $\mathbf{v}(t) = \mathbf{r}'(t)$, the speed is given by $|\mathbf{v}(t)|$, and the acceleration $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$

Practice Problems

1. A wagon is pulled a distance of 20m along a horizontal path by a constant force of 35N. The handle of the wagon is held at an angle of 30° above the horizontal. How much work is done?

2. Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (6, 3, -1)$, $\mathbf{v} = (0, 1, 2)$, and $\mathbf{r} = (4, -2, 5)$.

3. Find the equation of the plane containing the point $P(1, -2, 3)$ and the line of equation $\mathbf{r}(t) = (2 + 2t, -4t, 1 + t)$

4. Find the equation of the plane that contains the line $x = -1 - 2t$, $y = t$, $z = 5 + 3t$ and is perpendicular to the plane 3x-y+8z = 17.

5. Find the length of the curve $\mathbf{r}(t) = (e^t \cos t, e^t \sin t)$ for $0 \leq 1$

6. Find the unit normal vector, the unit tangent vector, and the curvature of the curve $\mathbf{r}(t) = (t, -t, 1 + t^2)$ at the point (1,-1,2).

7. A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\mathbf{i} + 12t^2\mathbf{j} - 6t\mathbf{k}$

8. A projectile is fired 200 m above the ground with an initial speed of 500 m/s and angle of elevation 30°. Find the maximum height reached.