

## Test 4 Review

### General Things

- 100 points possible
- NO CALCULATOR of any kind will be needed or allowed
- covers Chapter 13: Sections 13.1 - 13.7, Section 10.5, Section 12.6
- Extra Credit is available and will be due on Monday, December 3, 2007

### Section 13.1

- A **vector field** is a function  $\mathbf{F}$  that assigns to each point  $(x, y, z)$  a vector  $\mathbf{F}(x, y, z)$
- $\nabla f$  is really a vector field and is called a **gradient vector field**.

### Section 13.2

- Let  $C$  be a smooth curve defined by parametric equations

– Divide curve into subarcs and form Riemann sum.

$$- \int_C f(x, y) ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

- Let  $C$  be defined by the vector equation  $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$

$$- \int_C f(x, y, z) ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

$$- \int_C f(x, y, z) ds = \int_a^b f(\mathbf{r}(t)) |\mathbf{r}'(t)| dt$$

- Let  $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$  be a continuous vector field.

$$- \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt$$

### Section 13.3

- The Fundamental Theorem for Line Integrals

$$- \int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

– Evaluate the line integral of a conservative vector field, simply by knowing the value of  $f$  at the endpoints of  $C$ .

- If  $\int_C \mathbf{F} \cdot d\mathbf{r}$  is independent of path in  $D$ , then  $\mathbf{F}$  is a conservative vector field on  $D$ ; that is, there exists a function  $f$  such that  $\nabla f = \mathbf{F}$ .

- $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$  iff  $\mathbf{F}$  is a conservative vector field.

### Section 13.4

- The **positive orientation** of a simple closed curve  $C$  refers to a single *counterclockwise* traversal of  $C$ .
- **Green's Theorem**  $\int_C P dx + Q dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$
- Green's Theorem give the relationship between a line integral around a simple closed curve  $C$  and a double integral over the plane region  $D$  bounded by  $C$

### Section 13.5

- $\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left( \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \mathbf{i} + \left( \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \mathbf{j} + \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathbf{k}$
- If  $\mathbf{F}$  is conservative, then  $\text{curl } \mathbf{F} = \mathbf{0}$
- If  $\mathbf{F}$  is a vector field and  $\text{curl } \mathbf{F} = \mathbf{0}$ , then  $\mathbf{F}$  is a conservative vector field.
- The **divergence of  $\mathbf{F}$**  is defined by  $\text{div} \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

- Vector Forms of Green's Theorem

$$- \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_D (\text{curl} \mathbf{F}) \cdot \mathbf{k} dA$$

$$- \oint_C \mathbf{F} \cdot \mathbf{n} ds = \iint_D \text{div} \mathbf{F}(x, y) dA$$

### Section 10.5

- Let  $\mathbf{r}(u, v) = x(u, v) \mathbf{i} + y(u, v) \mathbf{j} + z(u, v) \mathbf{k}$  be a vector-valued function defined on a region  $D$  in the  $uv$ - plane.
- In general, a surface given as the graph of a function of  $x$  and  $y$ , that is, with an equation of the form  $z = f(x, y)$  can always be regarded as a parametric surface by taking  $x$  and  $y$  as parameters.
- tangent vectors

$$- \mathbf{r}_u = \frac{\partial x}{\partial u}(u_0, v_0) \mathbf{i} + \frac{\partial y}{\partial u}(u_0, v_0) \mathbf{j} + \frac{\partial z}{\partial u}(u_0, v_0) \mathbf{k}$$

$$- \mathbf{r}_v = \frac{\partial x}{\partial v}(u_0, v_0) \mathbf{i} + \frac{\partial y}{\partial v}(u_0, v_0) \mathbf{j} + \frac{\partial z}{\partial v}(u_0, v_0) \mathbf{k}$$

### Section 12.6

- The **surface area** of  $S$  is  $A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA$  where  $\mathbf{r}_u = \frac{\partial x}{\partial u} \mathbf{i} + \frac{\partial y}{\partial u} \mathbf{j} + \frac{\partial z}{\partial u} \mathbf{k}$  and  $\mathbf{r}_v = \frac{\partial x}{\partial v} \mathbf{i} + \frac{\partial y}{\partial v} \mathbf{j} + \frac{\partial z}{\partial v} \mathbf{k}$
- If  $S$  is  $z = f(x, y)$  then  $A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dA$

### Section 13.6

- The **surface integral** is defined as  $\iint_S f(x, y, z) dS = \iint_D f(\mathbf{r}(u, v)) |\mathbf{r}_u \times \mathbf{r}_v| dA$
- Any surface with equation  $z = g(x, y)$  has surface area
 
$$- \iint_S f(x, y, z) dS = \iint_D f(x, y, g(x, y)) \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} dA$$
- The **flux** of a vector field  $\mathbf{F}$  is  $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n} dS$
- If  $S$  is given by  $z = g(x, y)$  -  $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \left(-P \frac{\partial g}{\partial x} - Q \frac{\partial g}{\partial y} + R\right) dA$
- If  $S$  is given by vector function  $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA$

### Section 13.7

- **Stokes Theorem**  $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S}$