Test 3 Answer Key

① $y' = e^{3x+2y}$

$\frac{dy}{dx} = e^{3x}e^{2y}$

$\int e^{-2y}dy = \int e^{3x}dx$

$-\frac{1}{2}e^{-2y} = \frac{1}{3}e^{3x} + C$

$e^{-2y} = -\frac{2}{3}e^{3x} + C$

$-2y = \ln\left|-\frac{2}{3}e^{3x} + C\right|$

$\boxed{y = -\frac{1}{2}\ln\left|-\frac{2}{3}e^{3x} + C\right|}$

② $m(t) = m_0 e^{Kt}$

$m_0 = 100$; $m(5.24) = 50$

$50 = 100e^{K(5.24)}$

$\frac{1}{2} = e^{K(5.24)}$

$\boxed{K = \frac{\ln(\frac{1}{2})}{5.24} = -.1323}$

$m(t) = 100e^{-.1323t}$

$\boxed{m(20) = 100e^{\frac{\ln(\frac{1}{2})}{5.24}(20)} \text{ mg} = 7.09 \text{ mg}}$

③ $y'' - 6y' + 5y = 8e^x$

Char eqn: $r^2 - 6r + 5 = 0$

$(r-5)(r-1) = 0$

$r = 5 \quad r = 1$

$\boxed{y_c(x) = c_1e^{5x} + c_2e^x}$

$y_p(x) = Ae^x$ (solution to $y_c(x)$ ⇒ use modification rule)

$y_p(x) = Axe^x$

$y_p'(x) = Axe^x + Ae^x$

$y_p''(x) = Axe^x + 2Ae^x$

$y'' - 6y' + 5y = Axe^x + 2Ae^x - 6Axe^x - 6Ae^x + 5Axe^x = 8e^x$

$-4Ae^x = 8e^x \Rightarrow A = -2$ so $\boxed{y_p(x) = -2xe^x}$

solution: $\boxed{y(x) = c_1e^{5x} + c_2e^x - 2xe^x}$

④ $y'' + 4y' + 4y = x^2 - 3x$

Char eqn: $r^2 + 4r + 4 = 0$

$(r+2)(r+2) = 0$

$r = -2$

$\boxed{y_c(x) = c_1e^{-2x} + c_2xe^{-2x}}$

$y_p(x) = Ax^2 + Bx + C$

$y_p'(x) = 2Ax + B$

$y_p''(x) = 2A$

$y'' + 4y' + 4y = 2A + 4(2Ax + B) + 4(Ax^2 + Bx + C) = x^2 - 3x$

$= 4Ax^2 + (8A + 4B)x + (2A + 4B + 4C) = x^2 - 3x$

$\Rightarrow 4A = 1 \quad 8A + 4B = -3 \quad 2A + 4B + 4C = 0$

$A = \frac{1}{4} \quad 8(\frac{1}{4}) + 4B = -3 \quad 2(\frac{1}{4}) + 4(-\frac{5}{4}) + 4C = 0$

$B = -\frac{5}{4} \quad C = \frac{9}{8}$

$\boxed{y_p(x) = \frac{1}{4}x^2 - \frac{5}{4}x + \frac{9}{8}}$

So $\boxed{y(x) = c_1e^{-2x} + c_2xe^{-2x} + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{9}{8}}$

(5) $y'' + 2y' + 2y = 0 \quad y(0) = 0 \quad y'(0) = 10$

char eqn: $r^2 + 2r + 2 = 0$

$r = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2} = -1 \pm i$

$y(x) = e^{-x}(C_1 \cos x + C_2 \sin x)$

$y(0) = 0 \Rightarrow C_1 = 0$

$y(x) = e^{-x}(C_2 \sin x)$

$y'(x) = e^{-x}(C_2 \cos x) - (C_2 \sin x)e^{-x}$

$y'(0) = C_2 = 10$

$y(x) = 10e^{-x} \sin x$

(6)a $mx'' + cx' + kx = 0$

$3x'' + 30x' + 123x = 0$

$x(0) = 0 \; ; \; x'(0) = 2$

(b) $c^2 - 4mk = -576 < 0$

$\Rightarrow$ underdamped

(7)a $\lim_{n \to \infty} \frac{9^{n+1}}{10^n} = 9 \lim_{n \to \infty} \left(\frac{9}{10}\right)^n = 0$

since $\frac{9}{10} < 1$

$\Rightarrow$ convergence

(b) $\lim_{n \to \infty} \frac{n^3}{1+n^2} = \infty \Rightarrow$ Divergence