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9:03

MA241-005 Spring 2006 **Name**_____

TEST #2

Directions: Answer the following questions. You must **SHOW ALL WORK**. No credit will be given for answers only. You are allowed to use a nonprogrammable, non-graphing calculator. No cell phones, computers or PDAs.

1. Find the area of the region bounded by $y^2 = x$ and $x - 2y = 3$. **(12 pts)**

$x = y^2 \quad x = 3 + 2y$

$y^2 = 3 + 2y$

$y^2 - 2y - 3 = 0$

$(y - 3)(y + 1) = 0$

$y = 3 \quad y = -1$

$A = \left| \int_{-1}^{3} y^2 - 3 - 2y \, dy \right|$ or $A = \int_{-1}^{3} 2y + 3 - y^2 \, dy$

$= \frac{y^3}{3} - 3y - y^2 \Big|_{-1}^{3}$

$= \left(\frac{27}{3} - 9 - 9\right) - \left(-\frac{1}{3} + 3 - 1\right)$

$= -9 - (5/3) = \boxed{\frac{32}{3}}$

2. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$ and $y = x^2$ about the line $y = 1$. **(13 pts)**

$V = \pi \int_0^1 (1 - x^3)^2 - (1 - x^2)^2 \, dx$

$= \pi \int_0^1 (1 - 2x^3 + x^6) - (1 - 2x^2 + x^4) \, dx$

$= \pi \int_0^1 1 - 2x^3 + x^6 - 1 + 2x^2 - x^4 \, dx$

$= \pi \int_0^1 -2x^3 + x^6 + 2x^2 - x^4 \, dx$

$= \pi \left(-\frac{1}{2}x^4 + \frac{x^7}{7} + \frac{2x^3}{3} - \frac{x^5}{5}\right) \Big|_0^1$

$= \pi \left(-\frac{1}{2} + \frac{1}{7} + \frac{2}{3} - \frac{1}{5}\right) = \frac{23\pi}{210} \approx 7.23$

$y =$

3. Find the length of the curve $y = \frac{x^2}{2} - \frac{\ln x}{4}$ on the interval [2,4]. *Set up the integral, but do not evaluate it.* **(12 pts)**

$$\frac{dy}{dx} = x - \frac{1}{4}\left(\frac{1}{x}\right) = x - \frac{1}{4x}$$

$$L = \int_2^4 \sqrt{1 + \left(x - \tfrac{1}{4x}\right)^2}\, dx$$

4. Find the average value of the function $f(x) = t\sqrt{1+t^2}$ on the interval [0,5]. **(12 pts)**

$$f_{ave} = \frac{1}{5-0}\int_0^5 t\sqrt{1+t^2}\, dt$$

Let $u = 1+t^2$

$du = 2t\, dt$

$\frac{1}{2}du = t\, dt$

$$= \frac{1}{5}\int_0^5 \frac{1}{2}u^{1/2}\, du = \frac{1}{10}\cdot\frac{2}{3}u^{3/2}\Big|_0^5 = \frac{1}{15}(1+t^2)^{3/2}$$

$$= \frac{1}{15}(26^{3/2} - 1) \approx 8.77163$$

5. A force of 30 N is required to stretch a spring from its natural length of 30 cm to 40 cm. How much work would be done in stretching the spring from 40 cm to 50 cm? **(13 pts)**

$F = KX$

$30 = K(.10)$

$300 = K$

$F = 300x$

$$\int_{.10}^{.20} 300x = \frac{300x^2}{2} dx$$

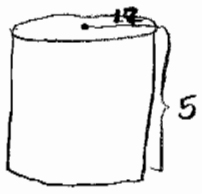
$$= 150x^2 \Big|_{.10}^{.20}$$

$$= 150(.2^2 - .1^2)$$

$$150(.03)$$

$$= 4.5 \text{ Joules}$$

6. A circular swimming pool has a diameter of 24 ft. and sides of height 5 ft. If the pool is completely filled with water, how much work is required to pump all of the water out over the side? (Use the fact that water weighs $62.5\, lb/ft^3$). *Set up the integral, but do not evaluate.* **(13 pts)**



$$V = \int_0^5 62.5\ \pi (12)^2 (5-y)\, dy$$

$$= \int_0^5 62.5 (144) \pi (5-y)\, dy$$

7. Sketch solution curves that pass through the following points: **(12 pts)**

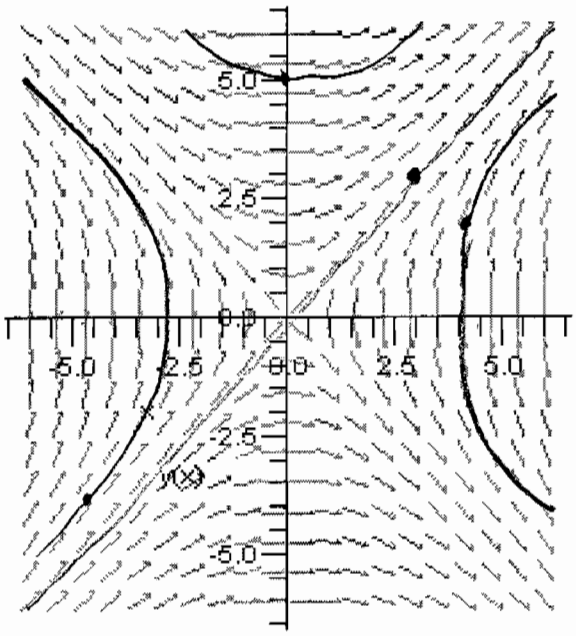
(a) y(0)=5

(b) y(3)=3

(c) y(4)=2

(d) y(8)=-4

y(-5)=-4



8. Given the differential equation $\frac{dy}{dx} = x^2 y$, $y(0) = 1$, use Euler's method with step size 1 to approximate $y(4)$. **(13 pts)**

$x_0 = 0 \quad y_0 = 1$

$x_1 = 1 \quad y_1 = 1 + 1(0^2(1)) = 1$

$x_2 = 2 \quad y_2 = 1 + 1(1^2(1)) = 2$

$x_3 = 3 \quad y_3 = 2 + 1(2^2(2)) = 10$

$x_4 = 4 \quad y_4 = 10 + 1(3^2(10)) = 100$