Test 3 Answer Key

1. $P(t) = P_0 e^{kt}$, $P_0 = 100$ $P(4) = 200$

a) $200 = 100 e^{k4}$

$2 = e^{k4}$

$\ln(2) = 4k$

$\boxed{k = \frac{1}{4}\ln(2)}$

$= .1732$

b) $P(t) = 100e^{\frac{1}{4}\ln(2)t}$

$P(t) = 100(2)^{\frac{1}{4}t}$

c) $P(20) = 100e^{\frac{1}{4}\ln(2)(20)}$

$= 3200$ units

2. a) $4y'' + y = 0$

$4r^2 + 1 = 0$

$r^2 = -\frac{1}{4}$

$r = \pm \frac{1}{2}i$

$\boxed{y(x) = C_1 \cos \frac{1}{2}x + C_2 \sin \frac{1}{2}x}$

b) $y'' - 6y' + 9y = 6xe^{2x}$

Char eqn:

$r^2 - 6r + 9 = 0$

$(r-3)(r-3)$

$r = 3$

$\boxed{y_c(x) = C_1 e^{3x} + C_2 x e^{3x}}$

$y_p(x) = (Ax+B)e^{2x}$

$y_p' = (Ax+B)\,2e^{2x} + Ae^{2x}$

$y_p'' = (Ax+B)\,4e^{2x} + 2e^{2x}(A) + 2Ae^{2x}$

$y'' - 6y' + 9y = 4Axe^{2x} + 4Be^{2x} + 2Ae^{2x} + 2Ae^{2x} - 12Axe^{2x} - 12Be^{2x} - 6Ae^{2x}$
$+ 9Axe^{2x} + 9Be^{2x} = 6xe^{2x}$

$\Rightarrow (4A - 12A + 9A)xe^{2x} + (4B + 2A + 2A - 12B - 6A + 9B) = 6xe^{2x}$

$A = 6$ $\quad B - 2A = 0 \Rightarrow B = 12$

$\boxed{y(x) = C_1 e^{3x} + C_2 x e^{3x} + (6x+12)e^{2x}}$ $\quad y_p(x) = (6x+12)e^{2x}$

3. $mx'' + cx' + kx = 0$

$m = 6$

$c = 18$

$k \Rightarrow F = kx$

$24 = k(2)$

$k = 12$

$6x'' + 18x' + 12x = 0$

$x'' + 3x' + 2x = 0$

$r^2 + 3r + 2 = 0$

$(r+2)(r+1)$

$r = -2 \quad r = -1$

$c^2 - 4mk \Rightarrow 18^2 - 4(6)(12) \Rightarrow$ $\boxed{\text{overdamped}}$

$x(0) = 0 \quad x'(0) = 7$

$x(t) = C_1 e^{-2t} + C_2 e^{-t}$

$x(0) = 0 \quad C_1 + C_2 = 0$

$-2C_1 - C_2 = 7$

$-C_1 = 7$

$C_1 = -7, \ C_2 = 7$

$x'(t) = -2C_1 e^{-2t} - C_2 e^{-t}$

$x'(0) = 7 \quad -2C_1 - C_2 = 7$

$\boxed{x(t) = -7e^{-2t} + 7e^{-t}}$

4. a) $a_n = \dfrac{n^2 - n + 7}{2n^3 + n^2}$

$\lim_{n\to\infty} \dfrac{n^2 - n + 7}{2n^3 + n^2}$

$\overset{H}{=} \lim_{n\to\infty} \dfrac{2n-1}{6n^2 + 2n}$

$\overset{H}{=} \lim_{n\to\infty} \dfrac{2}{12n+2} = \boxed{0 \text{ converges}}$

b) $a_n = \dfrac{\sin^2 n}{\sqrt{n}}$

$0 \le \dfrac{\sin^2 n}{\sqrt{n}} \le \dfrac{1}{\sqrt{n}}$

$\lim_{n\to\infty} 0 = 0 \quad \lim_{n\to\infty} \dfrac{1}{\sqrt{n}} = 0$

$\therefore$ $\boxed{a_n \text{ converges to } 0}$

c) $a_n = \dfrac{4^{n+1}}{3^{n+2}}$

$= \dfrac{4^1(4^n)}{3^2(3^n)} = \dfrac{4}{9}\left(\dfrac{4}{3}\right)^n$

Diverges by theorem since $\frac{4}{3} \not< 1$