MA-241
Test 4 Practice Problems

Determine whether the series is convergent or divergent using the test of your choice. Make sure you state the test used and all of the criteria needed.

1. $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$
2. $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{(n)}}{n+1}$
3. $\sum_{n=1}^{\infty} \frac{\cos 3n}{1+(1.2)^n}$

Determine whether the series is convergent or divergent by expressing it as a telescoping sum. If it is convergent, find its sum.

4. $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$

Find the radius of convergence and interval of convergence.

5. $\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{(n+2)!}$
6. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2 5^n}$

Find a power series representation for the function and determine the radius of convergence.

7. $f(x) = \frac{1}{x-5}$
8. $f(x) = \tan^{-1}\left(\frac{x}{3}\right)$

Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. Also find the associated radius of convergence.

9. $f(x) = \cos x$
10. $f(x) = e^{5x}$

Find the Taylor series for $f(x)$ centered at the given value of a.

11. $f(x) = \cos x, a = \pi$

Use a Maclaruin series derived in this section to obtain Maclaurin series for the given function.

12. $f(x) = \cos \pi x$
13. $f(x) = x^2 e^x$

Test 4 Practice Problems Answer Key

1. $\frac{n}{n^3+1} < \frac{n}{n^3} = \frac{1}{n^2}$ converges by the Comparison Test with the convergent p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$
2. $b_n = \frac{\sqrt{n}}{n+1} > 0, \{b_n\}$ is decreasing and $\lim_{n\to\infty} b_n = 0$ so the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\sqrt{n}}{n+1}$converges by the Alternating Series Test.
3. $|a_n| = \left|\frac{\cos 3n}{1+(1.2)^n}\right| \le \frac{1}{1+(1.2)^n} < \frac{1}{(1.2)^n} = \left(\frac{5}{6}^n\right)$, so $\sum_{n=1}^{\infty} a_n$ converges by comparison test with the convergent geometric series $\sum_{n=1}^{\infty} \left(\frac{5}{6}^n\right)$.
4. telescoping series that converges to $\frac{5}{6}$.
5. By the ratio test $R = \infty$ and $I = (-\infty, \infty)$
6. By the ratio test $R = 5$. Use p-series and Alternating Series Test to check endpoints and find the interval of convergence to be $I = [-5, 5]$.
7. $\sum_{n=0}^{\infty} -\frac{1}{5^{n+1}} x^n, R = 5, \mathrm{so} I = (-5, 5)$
8. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{3^{2n+1}(2n+1)} x^{2n+1}$ so $R = 3$.
9. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, R = \infty$
10. $\sum_{n=0}^{\infty} \frac{5^n}{n!} x^n, R = \infty$
11. $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{(2n)!} (x-\pi)^{2n}, R = \infty$
12. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!} x^{2n}, R = \infty$
13. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!} x^{n+2}, R = \infty$