White copy

4.(25 pts) For the function $f(x) = x^3 - x^2 - x$ find

a) the critical points

$f'(x) = 3x^2 - 2x - 1 = 0$, $x = \frac{2\pm\sqrt{(-2)^2-4(3)(-1)}}{3(2)} = 1, -1/3$

b) the intervals where $f(x)$ increases and where $f(x)$ decreases

$f'(x) = 3x^2 - 2x - 1 > 0$ on $(-\infty, -1/3)$ and $(1, +\infty)$, and thus $f(x)$ is increasing on $(-\infty, -1/3)$ and $(1, +\infty)$

$f'(x) = 3x^2 - 2x - 1 < 0$ on $(-1/3, 1)$ and thus $f(x)$ is decreasing on $(-2/3), 1)$

c) the relative maximum and minimum values of $f(x)$ if any

at $x = -1/3$ rel max value: $f(-1/3) = (-1/3)^3 - (-1/3)^2 - (-1/3) = 5/27$

at $x = 1$ rel min value: $f(1) = (1)^3 - (1)^2 - (1) = -1$

d) the intervals where $f(x)$ is concave up and where it is concave down

$f''(x) = 6x - 2$, $6x - 2 = 0$, $x = 1/3$

$f''(x) > 0$ and therefore $f(x)$ concave up on $(1/3, \infty)$

$f''(x) < 0$ and therefore $f(x)$ concave down on $(-\infty, 1/3)$

e) the inflection points if any: inflection point at $x = 1/3$: $(1/3, f(1/3)) = (1/3, -11/27)$

5.(10 pts) For the function $f(x) = 7x^3 + 3x^2 - 5$ find

a) $f'''(x)$

$f'(x) = 21x^2 + 6x$, $f''(x) = 42x + 6$, $f'''(x) = 42$

b)$f^{(15)}(x) = 0$

6. Let $f(x) = \frac{3x-1}{x-4}$

a) Find all horizontal asymptotes of $f(x)$.

$$\lim_{x\to\pm\infty} \frac{3x-1}{x-4} = \lim_{x\to\infty} \frac{3x}{x} = 3$$

$y = 3$ is the horizontal asymptote.

b) Find all vertical asymptotes $f(x)$. Evaluate one-sided limits of $f(x)$ to show the behavior of $f(x)$ near the vertical asymptotes.

$$\lim_{x\to 4^-} \frac{3x-1}{x-4} = -\infty, \qquad \lim_{x\to 4^+} \frac{3x-1}{x-4} = \infty$$

$x = 4$ is the vertical asymptote.

7. (10 pts) The following information about $y = g(x)$ is given:

i) $g(x)$ has two critical points $x = 1$ and $x = -1$ and ii) $g''(x) = \frac{8x(x^2+1)(x^2-3)}{(x^2+1)^4}$

Use the Second Derivative Test to find out if a relative maxima or minima is attained at $x = 1$.
$g''(1) = \frac{8(1)(1^2+1)(1^2-3)}{(1^2+1)^4} < 0$ then by the Second Derivative Test a rel. max occurs at $x = 1$

Blue Copy

4.(25 pts) For the function $f(x) = x^3 - x^2 - x - 5$ find
a) the critical points
$f'(x) = 3x^2 - 2x - 1 = 0$, $x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{3(2)} = 1, -1/3$
b) the intervals where $f(x)$ increases and where $f(x)$ decreases
$f'(x) = 3x^2 - 2x - 1 > 0$ on $(-\infty, -1/3)$ and $(1, +\infty)$, and thus $f(x)$ is increasing on $(-\infty, -1/3)$ and $(1, +\infty)$
$f'(x) = 3x^2 - 2x - 1 < 0$ on $(-1/3), 1)$ and thus $f(x)$ is decreasing on $(-2/3), 1)$
c) the relative maximum and minimum values of $f(x)$ if any
at $x = -1/3$ rel max value: $f(-1/3) = (-1/3)^3 - (-1/3)^2 - (-1/3) - 5 = 5/27 - 130/27)$
at $x = 1$ rel min value: $f(1) = (1)^3 - (1)^2 - (1) = -6$
d) the intervals where $f(x)$ is concave up and where it is concave down
$f''(x) = 6x - 2$, $6x - 2 = 0$, $x = 1/3$
$f''(x) > 0$ and therefore $f(x)$ concave up on $(1/3, \infty)$
$f''(x) < 0$ and therefore $f(x)$ concave down on $(-\infty, 1/3)$

e) the inflection points if any
inflection point at $x = 1/3$: $(1/3, f(1/3)) = (1/3, -146/27)$

5.(10 pts) For the function $f(x) = 7x^3 + 3x^2 - 5$ find
a) $f'''(x)$
$f'(x) = 21x^2 + 6x$, $f''(x) = 42x + 6, f'''(x) = 42$
b)$f^{(15)}(x) = 0$

6. Let $f(x) = \frac{3x-2}{x-5}$
a) Find all horizontal asymptotes of $f(x)$.

$$\lim_{x\to\infty} \frac{3x-2}{x-5} = \lim_{x\to\infty} \frac{3x}{x} = 3$$

$y = 3$ is the horizontal asymptote.
b) Find all vertical asymptotes $f(x)$. Evaluate one-sided limits of $f(x)$ to show the behavior of $f(x)$ near the vertical asymptotes.

$$\lim_{x\to 5^-} \frac{3x-2}{x-5} = -\infty, \qquad \lim_{x\to 5^+} \frac{3x-2}{x-5} = \infty$$

$x = 5$ is the vertical asymptote.
7. (10 pts) The following information about $y = g(x)$ is given:
i) $g(x)$ has two critical points $x = 1$ and $x = -1$
ii) $g''(x) = \frac{8x(x^2+1)(x^2-3)}{(x^2+1)^4}$
Use the Second Derivative Test to find out if a relative maxima or minima is attained at $x = -1$.
$g''(-1) = \frac{8(-1)((-1)^2+1)((-1)^2-3)}{((-1)^2+1)^4} > 0$ and then by the Second Derivative test a rel. min occurs at $x = -1$