Approximate Integration

Midpoint Rule:

$$\int_a^b f(x)\,dx \approx M_n = \Delta x\,[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)]$$

where

$$\Delta x = \frac{b-a}{n}, \qquad \bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$$

Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\,[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

where

$$\Delta x = \frac{b-a}{n}, \qquad x_i = a + i\Delta x, \qquad 0 \le i \le n$$

Simpson's Rule:

$$\int_a^b f(x)\,dx \approx S_n = \frac{\Delta x}{3}\,[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

where

$$\Delta x = \frac{b-a}{n}, \qquad x_i = a + i\Delta x, \qquad 0 \le i \le n$$

Error Bounds:

- Suppose $|f''(x)| \le K$ for $a \le x \le b$. If E_M and E_T are the errors in the Midpoint and Trapezoidal Rules, then

$$|E_M| \le \frac{K(b-a)^3}{24n^2} \qquad \text{and} \qquad |E_T| \le \frac{K(b-a)^3}{12n^2}$$

- Suppose $|f^{(4)}(x)| \le K$ for $a \le x \le b$. If E_S is the errors involved in using Simpson's Rule, then

$$|E_S| \le \frac{K(b-a)^5}{180n^4}$$