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# Math 141 - Continuity

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**Definition:** A function $f$ is *continuous at a number* $a$ if

$$\lim_{x \to a} f(x) = f(a) \ .$$

If $f$ is not continuous at $a$, we say $f$ is *discontinuous at* $a$.
This definition implicitly requires three thing if $f$ is continuous at $a$:

1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$ .

**Definition:** A function $f$ is *continuous from the right at a number* $a$ if

$$\lim_{x \to a^+} f(x) = f(a)$$

and $f$ is *continuous from the left at a number* $a$ if

$$\lim_{x \to a^-} f(x) = f(a) \ .$$

**Definition:** A function $f$ is *continuous on an interval* if it is continuous at every number in the interval. if $f$ id defined only on one side of an endpoint of the interval, we understand continuous at the endpoint to mean continuous from the left or continuous from the right.

**Theorem 0.1** *If $f$ and $g$ are continuous at $a$ and $c$ is a constant, then the following functions are also continuous at $a$:*

1. $f \pm g$
2. $fg$
3. $\dfrac{f}{g}$ *if* $g(a) \neq 0$
4. $cf$ .

***Theorem 0.2*** *The following types of functions are continuous at every point in their domain*

- *polynomials*
- *rational functions*
- *root functions*
- *trigonometric functions*
- *inverse trigonometric functions*
- *exponential functions*
- *logarithmic functions .*

***Theorem 0.3*** *If* $f$ *is continuous at* $b$ *and* $\lim_{x \to a} g(x) = b$ *, then* $\lim_{x \to a} f(g(x)) = f(b)$ *. In other words,*

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) \ .$$

***Theorem 0.4*** *If* $g$ *is continuous at* $a$ *and* $f$ *is continuous at* $g(a)$*, then the composite function* $f \circ g$ *given by* $(f \circ g)(x) = f(g(x))$ *is continuous at* $a$*.*

***Theorem 0.5 (The Intermediate Value Theorem)*** *Suppose that* $f$ *is continuous on the closed interval* $[a, b]$ *and let* $N$ *be any number between* $f(a)$ *and* $f(b)$*. Then there exists a number* $c$ *in* $(a, b)$*, such that* $f(c) = N$ *.*