Computational Methods in Economics and Finance
Homework 4

4.1. Consider the following functional equation

$$V(S) = f(S) + \beta \int V(z)p(z;S)dz$$

where f and p are known functions. Specifically, let $f(S) = S$ and $p(z;S)$ be Beta density function with $E[z|S] = (S + 0.5)/2$ and $Var[z|S] = 1/12 + (S - 0.5)^4$. Recall that the Beta density is

$$B(z;a,b) = \frac{z^{a-1}(1-z)^{b-1}}{B(a,b)}$$

and that $E[z] = a/(a+b)$ and $Var(z) = ab/((a+b)^2(a+b+1))$.

Write a Matlab script that computes and plots a polynomial approximation to V, with $\beta = 0.9$.

Note that there is a closed form solution to this problem but this should only be used to check your answer (if you want to use it at all).

4.2. Consider the following PDE

$$\rho V(S_1, S_2) = \mu S_1 \frac{\partial V(S_1, S_2)}{\partial S_1} + \frac{\partial V(S_1, S_2)}{\partial S_2} + \frac{\sigma^2}{2} S_1^2 \frac{\partial^2 V(S_1, S_2)}{\partial S_1^2}$$

subject to $V(S_1, T) = \max(K - S_1, 0)$, $V(0, S_2) = \exp(-\rho(T - S_2))K$ and $V(\infty, S_2) = 0$.

Solve this numerically using a spline approximation and the following parameter values: $\mu = 0.2$, $\sigma = 0.3$, $\rho = 0.1$, $K = 1$ and $T = 1$.

I suggest that the side condition for $s_1 = \infty$ be replaced by $\frac{\partial^2 V(\bar{S}_1, S_2)}{\partial S_1^2} = 0$ where $\bar{S}_1$ is a suitably large value. Also you will find you get better results if K is one of the breakpoints that define the spline function.

Check your solution by plotting the residual function. You can also check the error function if you know the closed form solution to the PDE.