Computational Methods in Economics and Finance
HW 1

1.1. Consider the function

$$C(u,v,a) = \exp\left(-\left((-\ln(u))^a + (-\ln(v))^a\right)^{1/a}\right)$$

for $u, v \in [0,1]$ and $a \in [1,\infty)$. The function has the property that it is increasing in both u and v and that $C \in [0,1]$. Other properties of this function are

$$\begin{aligned}
&C(u,v,1) = uv\\
&C(u,v,\infty) = \min(u,v)\\
&C(u,0,a) = C(0,v,a) = 0\\
&C(u,1,a) = u\\
&C(1,v,a) = v\\
&C(u,v,a) = C(v,u,a)
\end{aligned}$$

If one implements this function as written, numerical difficulties can arise as a gets large. Rewrite this function so it avoids this problem. Code this as a function in MATLAB. Be sure that it runs for compatible matrices of values of u and v (you can assume that a is scalar).

Test whether your implementation satisfies the above criteria for a grid of values of u and v and for a grid of values of a.

1.2. Continuous time stochastic processes are generally defined using stochastic differential equations such as

$$dS = \mu(S)dt + \sigma(S)dz$$

where μ and σ are functions of S and z is a standard Weiner process.

One way to think of such a process is as a limit of a discrete time process as the time increment between periods (Δ) goes to 0, leading to the approximation

$$S_{t+\Delta} \approx S_t + \mu(S_t)\Delta + \sigma(S_t)\sqrt{\Delta}e_t$$

where e_t is a standard Gaussian random variable (i.e., $e_t \sim N(0,1)$), with e_t independent of e_s for all $t \neq s$.

The simplest case of this (so-called Brownian motion) occurs when μ and σ are constants. Write a MATLAB function with the syntax
`S=brownian(mu,sigma,S0,Delta,n)`,
where S_0 is an initial value and n is the number of time steps. This function should accept 5 scalar arguments and return an $1 \times (n+1)$ vector (it should include S_0 as its first element).

You will need to use the function `randn` and will need to know how to define loops to write this function.

Write a script function that demonstrates the use of this function by generating 1000 time paths with $\mu = 0.1$, $\sigma = 0.2$, $S_0 = 1$, $\Delta = 0.01$ and $n = 500$. The script should also verify that the sample mean and variance over 1000 time paths are approximately equal to their expected values of $S_0 + \mu n\Delta$ and $\sigma^2 n\Delta$.

For extra credit, write the function so it accepts μ and S_0 as d vectors and σ as a $d \times d$ matrix and returns a $d \times (n+1)$ matrix. Write this so it doesn't loop over the d elements.