A Hotelling–Faustmann Explanation of the Structure of Christmas Tree Prices

Tomislav Vukina, Christiana E. Hilmer, and Dean Lueck

We examine the relationship between a tree price and a tree age (height) using a Hotelling–Faustmann type model of optimal plantation management, which accounts for the possibility of replanting and biological growth. The model predictions are tested using the data on Christmas tree prices in North Carolina collected in December 1997. The estimates show that, in general, the rates of change in prices between adjacent age cohorts reflect a competitive equilibrium in the capital market thus supporting the Hotelling–Faustmann paradigm.

Key words: capital theory, discrete time optimal control, uneven-aged forest management.

Rigorous empirical testing of the basic economic models of optimal natural resource pricing has proven to be difficult. The source of the difficulty has been the unavailability of micro-level data on either prices or extraction-harvest rates for in situ resource stocks.[1] For nonrenewable resources, Hotelling's famous pricing rule has rarely been tested with firm level data. Stollery's study of the International Nickel Company of Canada supported Hotelling's rule and Miller and Upton's study of oil company values supported a corollary called the *Hotelling Valuation Principle*. Most other studies, such as Halvorsen and Smith and Farrow, have refuted this bedrock of natural resource economics. In fact, two studies of old growth timber (essentially a nonrenewable resource) provide some of the most compelling support for the Hotelling model (Berck; Johnson and Libecap). Because standing timber is sold in the market, data are readily available for the in situ prices (stumpage prices) examined in the economic models. For renewable resources even fewer studies have been undertaken and complications with dynamics make empirical analysis even more treacherous.

In this article we examine the Christmas tree market with the objective to both theoretically and empirically investigate the relationship between tree price and tree age (height). We use the discrete time maximum principle to solve the problem of optimal infinite rotation plantation management. By explicitly accounting for the possibility of replanting and biological growth, we derive theoretical results describing the steady-state equilibrium price-age relationships. These results combine Hotelling's pricing rule with the well-known Faustmann optimal rotation formula. The possibility of replanting captures the site value effect and consequently increases the optimal rate of price change across adjacent age cohorts above the relevant interest rate. Biological growth introduces an adjustment to the standard Faustmann rule by decreasing the rate of optimal price change below the interest rate.

The market for Christmas trees has a number of interesting features that allow us to test the theory of optimal resource pricing. First, Christmas trees are sold without being processed so, not counting harvesting and transportation costs, tree prices are effectively in situ prices. Second, because con-

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[1] Krautkraemer provides an exhaustive survey of both the theoretical and empirical analysis on nonrenewables.

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sumers demand Christmas trees of different sizes they are harvested at different ages. Taller (and older) trees are more valuable so there is well-defined benefit of allowing trees to grow another year. Third, the most popular species of Christmas trees, Fraser Firs, grow roughly one foot per year, which establishes a straightforward relationship between price and age (height). Fourth, the demand for Christmas trees is highly seasonal with harvesting that occurs just once per year. Finally, Christmas trees are sold in highly competitive markets in which neither growers nor retailers have market power.

We conduct the empirical tests using prices from retail outlets in Raleigh, North Carolina and farm level prices received by growers in the western part of North Carolina collected in December 1997. The results indicate that, in general, the ratios of prices of adjacent age cohorts conform to Hotelling–Faustmann type pricing rules, reflecting a competitive equilibrium in the capital markets. In particular we are able to confirm two theoretical predictions. First, expressed on a per tree basis, prices across age cohorts increase at a rate higher than the interest rate indicating the presence of a significant site value effect. Second, the rate of change in price expressed on a per foot basis is smaller than the interest rate and increases with age (height) of the tree, the consequence of the declining growth rate as trees grow older.

The Christmas Tree Market and Price Data

Although Christmas trees come in a variety of species—firs, pines, junipers, cypresses, cedars and spruces—the focus of our analysis is Fraser fir (*Abies fraseri*). The Fraser fir is the most popular and the most valuable of Christmas tree species and is often referred to as the "Cadillac of Christmas trees." Its needles are flat, 0.5–1 inch long, dark green above and silver beneath. Fraser fir has excellent color and needle retention properties. Its strong branches are slightly turned up, which gives the tree a compact appearance. Little shearing is required to shape Fraser firs into Christmas trees. Fraser firs are grown only in the Appalachian Mountains, two hundred miles west of Raleigh, at elevations of 4,000–6,000 feet (Johnson).

Not counting the year needed for the site preparation, the production of Fraser firs involves a period of up to eight to ten years. The first year involves the planting of seedlings. During the time between planting and harvesting the stand is managed by pruning, shearing, mowing, fertilizing and spraying for pests. Harvesting typically begins in the fifth year and generally continues until the ninth year. Over the height range of concern, trees grow roughly one foot per year implying that height and age can be used interchangeably (Brown). Harvested trees are grouped into height classes based on one-foot intervals. The number of trees produced outside the height range of five to eight feet is very small. Trees shorter than five feet and taller than nine feet account for only 2% and 0.5% of the total number sold. The remaining sales are divided among 5-foot trees with 3%, 6-foot trees with 55%, 7-foot trees with 33.5% and 8-foot trees with 5% of the market (Brown). The small market for short trees is the result of the limited desirability of such trees and the competition from artificial and containerized trees. The similarly small market for tall trees is the result of a much smaller number of buildings with extremely high ceilings. Retail lots serving households generally do not sell trees used for decorating shopping malls, churches or office buildings.

The data on Christmas tree prices used in this study were collected in Raleigh, North Carolina during a three-week period prior to Christmas in 1997. Raleigh is a metropolitan area with over a half a million residents and hundreds of retail outlets making the Christmas tree market highly competitive. To examine the age–price relationship, ideally, one would want to use stumpage or farm level prices. Growers are, however, generally secretive about prices, revenues and production costs, therefore it was more convenient to collect data by directly surveying Christmas trees retailers servicing the household market.

We surveyed six types of retail outlets: garden centers, producer outlets, service clubs, individual entrepreneurs, chain stores and "choose-and-cut" operations.[2] Garden centers are year-round operations specializing in selling a broad spectrum of plants, trees, ornamentals and other nursery products and are considered the top of the high-end market. Producer outlets are seasonal outlets set

[2] Each outlet was surveyed four times. However, in this study we use only prices collected on the first survey weekend (December 3–6 1997).

up by the Christmas trees growers for the purpose of selling their own trees. Service clubs include churches, schools and similar organizations that are selling Christmas trees mainly for fund raising or charitable purposes. Individual entrepreneurs are typically small operators renting their retail lots on the fresh produce markets or busy traffic intersections. Chain retail stores include places like *Kroger, Winn-Dixie, Lowe's* and *Home-Depot* and are considered the bottom of the low-end market for Christmas trees. Choose-and-cut operations differ from the rest of the group because they do not cut their trees at the beginning of the season but rather let consumers pick and cut their own tree from a group of marked trees at the plantation. They regularly grow and sell a wide variety of different species. Since Fraser firs cannot be successfully grown near Raleigh, those firs sold by choose-and-cut operators are also cut and transported from western North Carolina, just like trees at the other outlets.

With the exception of choose-and-cuts, most other retailers specialize predominantly in Fraser firs and would only sporadically handle other species. All retailers price their trees in relatively homogeneous height/species groups with taller trees generally commanding higher prices. Prices are posted in dollars per tree and sellers do allow buyers to sort through the displayed inventory. We selected four different stores within each retailer group generating a cross-section of twenty-four retail outlets and collected prices for six sizes of trees (4–5, 5–6, 6–7, 7–8, 8–9-feet, and 9-feet and higher) and two quality grades (No. 1—higher quality and No. 2—lower quality).[3]

In addition to recording the retail level prices, we also asked retailers about prices they paid to their suppliers. The number of observations for farm level prices that we collected was relatively small (six sets of Fraser fir prices for all heights) because retailers were somewhat hesitant to reveal prices they paid to the growers. For chain retail stores the procurement of trees is typically done through their regional headquarters and hence local stores managers did not know the farm-level prices of their trees. Therefore, as explained more precisely later, empirical analysis of the Fraser fir pricing structure was performed using both farm level (stumpage prices) and retail level data.

A Hotelling–Faustmann Age–Price Model

The most prominent result in the economics of natural resources—Hotelling's rule—which Solow calls "the fundamental principle of the economics of exhaustible resources" states that in equilibrium the value of the resource stock must be growing at the rate of interest. The reason for this relationship among prices over time is that the producer of a natural resource extracts the resource in two sequential time periods. Since it is optimal to do so, it must be the case that the price increases by an amount sufficient to offset the interest lost from delaying the extraction by one period.

The parallel result in the forestry economics, known as the Jevons-Fisher formula (Samuelson), states that in a single rotation case, an even-age plantation should be harvested when the rate of growth of the undiscounted net benefits from harvesting equals the rate of interest. Because the single rotation model ignores the opportunity cost of land, the formula yields the correct competitive rotation period only if land is so abundant as to be rent-free (Samuelson). To compensate the owner for the opportunity cost of land, the price of a tree must grow at a rate higher than the interest rate. If the price was expected to increase only at the rate of interest, the owner could sell trees today, sell the land to somebody willing to replant the new trees and invest the proceeds at the market rate of interest. This would yield greater return than if he held the trees until the next year (Lyon). This is a qualitative description of the famous Faustmann solution to the infinite cycle optimal rotation problem.

Like an exhaustible resource and unlike the standard Faustmann model of forestry, Christmas trees are also optimally harvested at more than one age. The Christmas tree market is characterized by the harvest and sale of many different sized (and therefore aged) trees from the same stand in the same year; that is, some trees are harvested when they are six years old, some when they are seven years old and so on. The producer has

[3] The only quantifiable quality factor is the "taper" (the ratio of the base diameter of a tree to its height). A taper of 60% is considered the lower limit for a high quality tree. However, there are other factors that influence quality such as defects, holes and poor shape (Brown). Since sellers are not consistent about categorizing their trees and since nearly all trees are sold as No. 1, we chose not to use the data on No. 2 category trees.

a choice of the age (size) at which to harvest each individual tree. Older trees receive higher prices. In order for it to be meaningful to harvest trees from many age cohorts, the interest income lost and the opportunity to plant a new tree deferred by waiting for a tree to grow one year older must be exactly balanced out by the increase in the price of the tree. The capital market equilibrium rule implies a relationship between prices for different age trees. In this section we formally develop this relationship between prices.

We model the decision process of a representative competitive Christmas tree producer as an infinite horizon, discrete time optimal control problem. We assume that each grower maximizes an infinite stream of discounted profits by selling trees from a fixed parcel of land entirely devoted to the Christmas tree production in perpetuity. The representative firm's inter-temporal profit function is:

$$(1)\quad \Pi = \sum_{t=0}^{\infty} \delta^t \left[\sum_{k=\hat{k}}^{K} P(k,t) q(k,t) \right]$$

where $t = 0, 1, 2, \ldots, \infty$ is the year index, $k = 0, 1, 2, \ldots, \hat{k}, \ldots, K$ is a cohort index with $\hat{k}$ denoting the first commercially harvestable cohort and K denoting the oldest commercially harvestable cohort. $P(k,t)$ represents the market price per tree of cohort k in time t net of replanting, maintenance and marketing costs with $P(k,t) = 0$ for $k = 0, 1, 2, \ldots, \hat{k}-1$. The number of trees of the kth cohort harvested in year t is denoted by $q(k,t)$. $\delta = 1/(1+r)$ is a discount factor with r representing the rate of interest.

The firm maximizes equation (1) subject to a replanting constraint, aging dynamics and a set of feasibility constraints. Denoting the number of standing trees of cohort k in year t by $x(k,t)$, the replanting constraint states that trees of any age group k harvested in year t will be replanted and will become age-zero cohort trees in the same year, so that:

$$(2)\quad x(0,t) \le \sum_{k=\hat{k}}^{K} q(k,t)$$

The aging dynamics constraint states that trees not harvested in the current year will grow to become one-year older trees next year:

$$(3)\quad x(k+1, t+1) \le x(k,t) - q(k,t)$$

and the feasibility constraints are:

$$(4)\quad q(k,t) \le x(k,t), \quad q(K,t) = x(K,t)$$

$$(5)\quad x(k,t) \ge 0, \quad q(k,t) \ge 0$$

The first part of equation (4) states the fact that one cannot harvest nonexisting trees whereas the second part of equation (4) is a maximum useful age condition stating that all trees that reach the final marketable age K in a given year must be harvested in the same year. The constrained maximization problem can now be written in the form of a Lagrangean function:

$$(6)\quad L = \sum_{t=0}^{\infty} \left\{ \begin{array}{l} \delta^t \sum_{k=\hat{k}}^{K} P(k,t) q(k,t) + \delta^t \lambda(0,t) \\ \left[\sum_{k=\hat{k}}^{K} q(k,t) - x(0,t) \right] \\ + \sum_{k=0}^{K-1} \delta^{t+1} \lambda(k+1, t+1) \\ \left[x(k,t) - q(k,t) \right. \\ \left. - x(k+1, t+1) \right] + \delta^{t+1} \\ \lambda(K+1, t+1) \\ \left[x(K, t+1) - q(K, t+1) \right] \end{array} \right\}$$

Notice that the inequality constraint in equation (4) is redundant as it is already covered by the aging constraint in equation (3) and the nonnegativity constraints in equation (5). Therefore only the equality constraint for $k = K$ from equation (4) explicitly appears in the Langrangean. The complete set of Kuhn–Tucker conditions for the optimization of equation (6) includes:

$$(7)\quad P(k,t) + \lambda(0,t) - \delta\lambda(k+1, t+1) \le 0, \quad [P(k,t) + \lambda(0,t) - \delta\lambda(k+1, t+1)] q(k,t) = 0$$

$$(8)\quad \delta\lambda(k+1, t+1) - \lambda(k,t) \le 0, \quad [\delta\lambda(k+1, t+1) - \lambda(k,t)] x(k,t) = 0$$

$$(9)\quad \sum_{k=\hat{k}}^{K} q(k,t) - x(0,t) \ge 0$$

$$(10)\quad x(k,t) - q(k,t) - x(k+1, t+1) \ge 0$$

$$(11)\quad x(K,t) - q(K,t) = 0$$

$$(12)\quad x(k,t) \ge 0, \quad q(k,t) \ge 0, \lambda(k,t) \ge 0$$

The existence of the maximum is guaranteed by the continuity of the profit function and the requirement that constraints define a compact set.[4]

The first-order condition, equation (7), describes the flow equilibrium on the market for cut Christmas trees. Since trees are cut at each of the ages ($k = \hat{k}, \ldots, K$), for all those cohorts the first part of equation (7) holds with equality.[5] The optimality condition requires that the harvest be arranged such that the price of a given cohort in a given period $P(k, t)$ equals the discounted value of an additional tree in the next oldest cohort, one year into the future $\delta\lambda(k + 1, t + 1)$ minus the shadow price of a new seedling $\lambda(0, t)$. An additional tree of age k harvested this year will reduce the stock of cohort $(k + 1)$ in period $(t + 1)$ but will also free up the land such that a new seedling can be replanted. For nonharvestable cohorts ($k = 0, 1, \ldots, \hat{k} - 1$), the first part of equation (7) holds with inequality indicating that when $q(k, t) = 0$ the market price of a tree has to be less than the discounted shadow price of the next oldest cohort corrected for the shadow price of a new seedling hence it makes sense to wait for trees to grow taller.

The first-order condition, equation (8), defines the stock equilibrium on the live Christmas tree capital market by describing the behavior of the shadow prices of adjacent cohort stocks. In the meaningful case where $x(k, t) > 0$, the first part of equation (8) holds with equality thus merely restating the Hotelling rule, which says that in a competitive equilibrium the shadow price of a living tree (from one cohort to the next) has to increase at the rate of interest.

Because we are concerned with the behavior of competitive prices across age (size) groups *within the same period* rather than over time, a steady-state equilibrium is a natural assumption. A steady state is characterized by unchanging market prices, stocks, harvests and shadow prices.[6] Specifically,

$$(13)\quad P(k, t) = P(k), \quad x(k, t) = x(k), \quad q(k, t) = q(k), \quad \lambda(k, t) = \lambda(k)$$

Using equation (13), the interior solution part of equation (8) yields $\lambda(0) = \delta^{k+1}\lambda(k + 1)$ which is then substituted into the expressions for $P(k)$ and $P(k - 1)$ from equation (7) to obtain an analytical solution for the rate of change in net price between any two adjacent marketable cohorts:

$$(14)\quad \frac{P(k) - P(k - 1)}{P(k - 1)} = \frac{\delta - 1}{\delta^k - \delta}$$

It turns out, perhaps not surprisingly, that (14) is in fact a discrete time version of the Faustmann formula, which can be seen after rewriting equation (14) as:

$$(15)\quad \frac{P(k) - P(k - 1)}{P(k - 1)} = \frac{r}{1 - \delta^{k-1}} = r + \frac{r}{\delta^{1-k} - 1}$$

It is immediately obvious that the rate of change in price exceeds the interest rate by a factor representing the site value. When the owner of the stand foregoes harvesting a tree this year, he not only forgoes the current income but he also foregoes the replanting that could be started and therefore must be compensated for both. He is compensated for the foregone income by having the net price increase at the rate of interest r and he is also compensated for the delay in replanting by having the price increase at the rate $r/(\delta^{1-k} - 1)$. The site value effect is more pronounced with younger cohorts, whereas for very large *k's* outside the harvestable region, the site value effect becomes gradually negligible. One can think of equation (15) as a modified version of the Hotelling pricing rule where the modification is of the Faustmann type. The results can be summarized as:

Proposition 1. *The rate of change in the* net price per tree *between two adjacent age groups is greater than the interest rate and is decreasing with the age of the tree.*

[4] This approach was inspired by Conrad's multiple cohort clam fishery paper. A useful discussion of the Kuhn–Tucker conditions in the linear programming harvest scheduling problems is found in Berck and Bible.

[5] In the usual forestry model such as Johnson and Scheurman, there is a unique $q(k, t)$ for which (7) holds with equality and all other $q(k, t)$ are zero.

[6] The steady-state assumption is reasonable. According to Pat Wilkie, executive director of the North Carolina Christmas Tree Association, both retail and farm level prices of Christmas trees have been flat over the last decade (Boone). The relatively stable prices are due to a fairly predictable aggregate demand (one tree per family) and the fact that weather patterns exert a relatively minor influence on the production of Christmas trees.

Additional implications can be derived by examining the model with prices expressed on a per foot basis. The grower's profit function needs to be changed to reflect the fact that prices $p(k, t)$ are now expressed on a per foot basis rather than on a per tree basis:

$$(16)\quad \pi = \sum_{t=0}^{\infty} \delta^t \left[\sum_{k=k}^{K} p(k, t)h(k)q(k, t) \right]$$

The height of a tree of age k is governed by $h(k) = (1 + \gamma_{k-1})h(k - 1)$ where γ represents the percentage growth rate. Because Christmas trees grow by a constant annual amount (for all marketable sizes, Fraser fir grows roughly one foot per year), the growth rate is actually inversely related to the age of the tree, so $\gamma_k = 1/k$. The Kuhn–Tucker conditions for the maximization of equation (16) subject to the previously employed set of constraints, after some reasonably straightforward algebra, yield:

$$(17)\quad \frac{p(k) - p(k-1)}{p(k-1)} = \frac{1}{1 + \gamma_{k-1}} \left[\frac{r}{1 - \delta^{k-1}} - \gamma_{k-1} \right]$$

Notice that the first element in the square bracket on the right-hand-side of equation (17) is the Faustmann formula for the optimal per tree price change from equation (15) and the second element is the percentage growth factor. The overall effect is due to the negative impact that the biological growth exerts on the rate of change in price per foot. Since the compensating effect of growth is diminishing in older cohorts, the rate of change in price per foot becomes larger for older cohorts. In effect, tree growth compensates the grower for the "carrying charges" of forgoing earlier revenue from younger, smaller trees. The results can be summarized as:

PROPOSITION 2. *The rate of change in the* net price per foot *between two adjacent height groups is smaller than the interest rate and is increasing with the age of the tree.*

We prove both propositions by inspection. As k increases, from equation (15) it is easily seen that the rate of optimal per tree price change is approaching the interest rate from above, and from equation (17) that the rate of optimal per foot price change is approaching the interest rate from below.

Both versions of the Hotelling–Faustmann pricing rules are strictly supply side phenomena and they should always hold regardless of the demand conditions. However, there are infinitely many prices that satisfy those rules. Bringing the demand side into the picture would enable the determination of the unique price level. Since our objective is not to derive the dynamic equilibrium price trajectory but to test the steady-state price–age relationship across age cohorts, the full description of the intertemporal competitive market equilibrium is not required. One possible market equilibrium with only four age cohorts is depicted in figure 1. On the demand side, we maintain the assumption that demand does not shift over time and incorporate the fact that consumer demand is greatest for the middle aged cohorts and smallest for the youngest and oldest cohorts. On the supply side, growers expect the price of Christmas trees between adjacent cohorts to rise at the Hotelling–Faustmann rate, equation (15), and their expectations are indeed fulfilled. Competitive conditions prevail and market clears in each cohort.

Empirical Evidence

Testing Hotelling–Faustmann proposition would ideally require in situ (stumpage) prices net of production and harvesting costs. Our data set includes the combination of farm level and retail level prices for different age (height) trees all coming from a single cross-sectional survey. Since the farm level data set is fairly small, it was necessary to use the larger retail level data set as well. Another issue is the fact that the empirically observed cost of production data for Christmas tree plantation is not available.[7]

The first practical problem becomes how to convert the retail-level prices into farm-level (stumpage) prices; that is, how to purge the retail prices of transportation and other retailing costs.[8] To do this, we assume that at the beginning of the Christmas marketing season (i.e., on the first survey date), the relationship between the retail-level and farm-level (stumpage) prices is determined by a constant percentage retail markup α given by $P_{k,i} = (1 + \alpha_{k,i})F_{k,i}$, where $P_{k,i}$ denotes

[7] The only source of cost data is the extension enterprise budgets for Christmas tree growers (Hamilton, Eickhoff, and McKinley; Safley). These budgets are constructed based on the fixed harvesting schedule and a constant per acre cost of establishing a plantation.

[8] For a literature survey on marketing margins, see Wohlgenant.

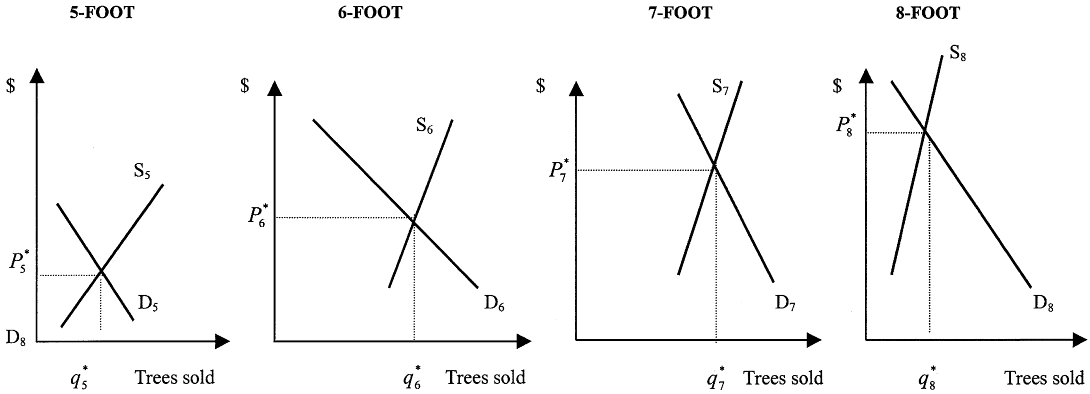

Figure 1. Competitive Equilibrium in 4-Cohort Tree Market

a retail and $F_{k,i}$ a farm level (stumpage) price of a Christmas tree of age-height cohort k and retail outlet i. A constant percentage markup assumption makes sense because transportation and setup costs are roughly proportional to the height of the trees (Johnson). Under these conditions the rate of change in the retail-level price equals the rate of change in the farm-level price: $(P_{k+1} - P_k)/P_k = (F_{k+1} - F_k)/F_k$ because the constant percentage retail markup cancels out and any systematic difference in retail pricing strategies among different outlets and different size trees (i.e., the difference in the α terms) would have no impact on the rates of change in stumpage price across heights. A statistical test below shows that the assumption of constant percentage retail markup cannot be rejected. This allows us to use the retail-level prices in testing the propositions about stumpage prices.

Purging the farm level prices from production cost is substantially more complicated because costs of production occur in different time periods.[9] All we can do is work with the observed market prices and speculate how the presence of annual (marginal) costs might influence the derived results.

Finally, our analysis requires some information about relevant interest rates. The interest rates that the Christmas tree sector in North Carolina faced in 1997 was obtained from the Mountain Farm Credit in Asheville, NC, through personal communication. The typical interest rate on a fixed rate one-year operating loan in 1997 was between 9% and 10%, whereas long-term renewable loan interest rates were 1–2 percentage points higher. Thus, we chose 11% as a reasonable approximation for the industry relevant interest rate for the 1997 growing season.

Each of the above two propositions is tested two ways. The first method is based on the comparisons of means. These results are presented in tables 1 and 2. The second method estimates a relationship between prices and heights to obtain the in-sample forecasts of rates of price change. Because we want the data to determine the correct functional form describing the relationship between prices and heights, we use the Box–Cox estimation procedure. These results are presented in table 3.

Comparison of Means

Table 1 presents the results of the analysis carried out with prices expressed on a per tree basis. In the first section of table 1 we present the means of the retail level prices for different age groups of Fraser firs. As expected, these numbers clearly show that the prices are rising as the age of the tree increases. To test proposition 1 [equation (15)] we need the rates of change in prices between adjacent cohorts. To calculate those we must observe the price of a tree of a given height at a given store and a

[9] The effect of the cost structure on the predicted rate of change in the net price can be easily demonstrated using a simple single rotation, even-aged plantation profit maximization model: Max $\pi = \delta(P_1 - C_1)q_1 + \delta^2(P_2 - C_2)q_2 - E_0$, with E_0 indicating the establishment cost. The first order conditions generate the following Hotelling type result: $(P_2 - P_1)/P_1 = r + (C_2 - (1+r)C_1)/P_1$ which indicates that the percentage change in the net price of a two-year old tree over the price of a one-year old tree equals the rate of interest plus the cost correction factor. If the marginal cost of the second cohort is equal to the compounded marginal cost of the first cohort, the standard Hotelling result obtains. If however $C_2 > (1 + r)C_1$, then the rate of increase in the net price between the two adjacent age groups will exceed the rate of interest and vice versa.

Table 1. Fraser Fir Prices Summary Statistics: Price Per Tree

Retail Prices						
Price per Tree						
Tree height	9+ feet	8 feet	7 feet	6 feet	5 feet	4 feet
Observations	19	22	22	23	18	12
Mean	86.665	67.512	50.367	40.558	33.026	27.164
Standard error	(7.38)	(4.69)	(2.33)	(1.87)	(1.44)	(1.10)
Rate of Change in Price per Tree						
Tree height	Total	5–6 feet	7–8 feet	8–9 feet	4–5 feet	6–7 feet
Observations	92	18	21	19	12	22
Mean	0.279	0.319*	0.304*	0.294*	0.239*	0.233*
Standard error	(0.023)	(0.069)	(0.042)	(0.066)	(0.044)	(0.033)
		[—	—	—	—	—]
Farm Level Prices						
Price per Tree						
Tree height	9+ feet	8 feet	7 feet	6 feet	5 feet	4 feet
Observations	4	6	6	6	6	5
Mean	54.988	34.658	25.325	20.658	16.242	12.150
Standard error	(5.20)	(1.23)	(0.33)	(0.33)	(0.40)	(0.70)
Rate of Change in Price per Tree						
Tree height	Total	8–9 feet	7–8 feet	4–5 feet	5–6 feet	6–7 feet
Observations	27	4	6	5	6	6
Mean	0.144	0.596	0.368*	0.322*	0.274*	0.226*
Standard error	(0.015)	(0.163)	(0.040)	(0.062)	(0.023)	(0.012)
			[—	—	—	—]

Notes: * Indicates the rate of change in price is significantly greater than an interest rate of 11% at a 5% significance level using t-test.
Differences in means are tested with a Duncan's new multiple comparison test.
Means connected with a line are not significantly different.

tree one foot taller.[10] For example, the rate of change in price for a 4–5-foot tree is found by subtracting the price of a 4-foot tree from the price of a 5-foot tree and dividing by the price of a 4-foot tree. The exact number of observations used in the calculations of various means are listed in the tables.

The testing is accomplished in two steps. First, we test that the rates of change in price per tree for all heights are greater than the interest rate, and second that the rates of change are decreasing as age/height increases. The first test was accomplished using an 11% interest rate. A simple t-test rejected the hypothesis that the rates of change in retail price per tree (second section in table 1) are equal to 11% over all heights thus supporting the first part of proposition 1. The same is true for the farm level prices (bottom section in table 1) except for the 8–9-feet trees where the standard error (0.163) is so large that the hypothesis could not be rejected.

To assess cross-mean differences across heights we use Duncan's new multiple range test. This test is more appropriate than the standard pair-wise t-test because it provides the correct significance levels when testing the differences among multiple means with the pair-wise comparisons not specified a priori (Steel et al., p. 194). The main feature of Duncan's test is that critical values depend on the number of positions between the two means being compared, hence the means have to be ordered. Our tables are organized such that the means are displayed in decreasing order from left to right. The highest average rate of change in price per tree is roughly 32% while the lowest is 23% (see the second section in table 1). A line that connects entries indicates that the rates of change in prices for different heights are

[10] The necessary observations to calculate the percentage change in price did not exist for nine feet and taller trees and hence the rates of change for the 9+ feet trees were not calculated. Some additional observations had to be dropped for trees that lacked comparable price observations.

Table 2. Fraser Fir Prices Summary Statistics: Price Per Foot

Retail Prices						
Price per Foot						
Tree height	9+ feet	8 feet	7 feet	4 feet	6 feet	5 feet
Observations	19	22	22	12	23	18
Mean	9.628	8.439	7.195	6.791	6.760	6.605
Standard error	(0.105)	(0.086)	(0.086)	(0.169)	(0.086)	(0.144)
Rate of Change in Price per Foot						
Tree height	Total	8–9 feet	7–8 feet	5–6 feet	6–7 feet	4–5 feet
Observations	92	19	21	18	22	12
Mean	0.069	0.150	0.141	0.099	0.064*	−0.027*
Standard Error	(0.045)	(0.011)	(0.010)	(0.011)	(0.009)	(0.017)
		[——	——	——	——]	
				[——	——	——]
Farm Level Prices						
Price per Foot						
Tree height	9+ feet	8 feet	7 feet	6 feet	5 feet	4 feet
Observations	4	6	6	6	6	5
Mean	6.110	4.332	3.618	3.443	3.248	3.038
Standard error	(0.578)	(0.154)	(0.047)	(0.055)	(0.080)	(0.174)
Rate of Change in Price per Foot						
Tree height	Total	8–9 feet	7–8 feet	5–6 feet	4–5 feet	6–7 feet
Observations	27	4	6	5	6	6
Mean	0.032	0.419	0.197	0.067	0.057	0.051*
Standard error	(0.034)	(0.144)	(0.037)	(0.021)	(0.045)	(0.010)
			[——	——	——	——]

Notes: *Indicates the rate of change in price is significantly less than an interest rate of 11% at a 5% significance level using t-test.
Differences in means are tested using Duncan's new multiple comparison test.
Means connected with a line are not significantly different.

not significantly different from each other which does not support the second part of proposition 1.[11] One possible explanation of this result is that the diminishing importance of the site effect as the age of a tree increases is dampened by the asymmetric cost structure (see footnote 9). Namely, there is some evidence that the annual (marginal) cost per tree of a given harvestable cohort is larger than the compounded annual per tree cost of a one year younger cohort (Safley). This may explain why we do not see price rates decreasing with the age of the tree.

It is interesting to note that both retail level and farm level prices generate very similar results thereby indirectly supporting the constant percentage retail markup assumption. We tested this result two different ways. First, we conducted a test to see whether farm level average price rates in different size groups are jointly different from retail level average price rates and found that they are the same. Second, we conducted a pair-wise test of the mean price rates for each height (age) cohort and found a significant difference only for the tallest category of trees at the 5% but not at the 10% significance level.

Summary statistical evidence relevant for proposition 2 [equation (17)] is presented in table 2. This first and third sections present the means of the retail and farm level prices for Fraser firs expressed on a per foot basis whereas the second and the fourth sections present the average rates of change in those prices.[12] Similar to proposition 1, the testing

[11] To test the difference between rates of change in price per tree between, say 5–6-foot and 8–9-foot trees which are one mean apart, the margin of sampling error is obtained by multiplying the total standard error (0.023) by the critical value (3.12) obtained from Duncan's table (Steel et al., p. 620). The product is then added and subtracted from the 5–6-foot mean to obtain the appropriate confidence interval (0.319−0.072, 0.319+0.072). Because this interval contains the 8–9-foot trees mean of 0.294, we conclude that the two means are not significantly different.

[12] This table constructs price per foot by dividing price by the lower limit of the foot interval (e.g., the price per foot for an 8–

Table 3. Estimation Results: Box–Cox Model

Dependent Variable: Price per Tree

Variable	Coefficient Estimate	Standard Error	T Value
Intercept	2.4940	0.4781	5.215
Height	0.1819	0.0702	2.591
Retail Dummy	1.4898	0.8657	1.721
λ	1.5878	0.3234	4.911
θ	0.2305	0.4781	1.222

	Predicted Rate of Price Change per Tree	Standard Error	T Values for Adjacent Rates of Change
4–5 Feet	0.2643*	0.0241	+0.5173
5–6 Feet	0.2789*	0.0147	−1.2991
6–7 Feet	0.2553*	0.0106	+1.0164
7–8 Feet	0.2696*	0.0092	+2.6594
8–9 Feet	0.3002*	0.0069	

Dependent Variable: Price Per Foot

Variable	Coefficient Estimate	Standard Error	T Value
Intercept	0.9051	0.0011	802.32
Height	0.0525	0.0001	467.53
Retail Dummy	0.9389	0.0024	391.94
λ	1.2626	0.0007	1860.8
θ	0.1936	0.0012	157.5

Elasticity	Predicted Rate of Price Change per Foot	Standard Error	T Values for Adjacent Rates of Change
4–5 Feet	0.0963	0.0099	+0.1922
5–6 Feet	0.0988	0.0084	−2.4590
6–7 Feet	0.0711**	0.0075	+0.6982
7–8 Feet	0.0785**	0.0075	+2.6117
8–9 Feet	0.1048	0.0067	

Notes: λ transforms the independent variable (height) while θ transforms the dependent variable (price or price per foot).
* Indicates the rate of change in price is significantly greater than an interest rate of 11% at a 5% significance level.
** Indicates the rate of change in price is significantly less than an interest rate of 11% at a 5% significance level.

is also accomplished in two steps. First, we test that the rates of change in price per foot for all heights are smaller than the interest rate, and second that the rates of change are increasing as age/height increases.

A t-test of the rates of change in retail level prices against an industry relevant interest rate of 11% reveals that only the 6–7 year and 4–5 year price changes are significantly smaller than the interest rate. A t-test using the farm level prices reveals that the rate of change for the 6–7-foot tree is significantly smaller than the interest rate while the other four cohorts are not significantly different from the interest rate. The empirical evidence in support of the first part of proposition 2 based on these tests is therefore ambiguous.

To assess cross-mean differences among rates of change of price per foot across heights we again use Duncan's test. The retail price rates of change reveal two peculiar results. First, the behavior of the rate of change in the retail price per foot is not monotonic: the rate of change for the 5–6-foot trees is larger than for 6–7-foot trees. Second, the rate of change in per foot price for 4–5-foot trees is negative (−2.7%). As seen from equation (17), for any meaningful

9-foot tree is the price divided by 8). Similar results are obtained when the calculation is made using midpoint or the upper limit of the interval.

parameter values, the negative rate of change in price per foot and nonmonotonicity are not possible. However, looking at the significance of the differences between means, we observe that the tallest four categories and the shortest three categories of trees have means that are insignificantly different from each other, downplaying the importance of both unusual results. What is left is the finding, unaffected by the anomalous behavior of price rates, that 8–9 and 7–8-foot trees have significantly larger rates of change in price per foot than the 4–5-foot trees providing some support for the second part of proposition 2. The results obtained using farm level prices exhibit the nonmonotonicity problem as well. However, all rates of change in price per foot except for the tallest tree category are not significantly different from each other thus minimizing the importance of these anomalous results. The fact that the tallest trees exhibit a significantly higher rate of change in price per foot than the other smaller tree categories lends some support for the second part of proposition 2.

Box–Cox Regression Analysis

Our second method to testing propositions 1 and 2 is to econometrically estimate the relationship between prices and heights. Because we want the specific functional form for the relationship between price and height to be determined by the data, we use the Box–Cox transformation on both the dependent (price) variable and the independent (height–age) variable (see Spitzer, Greene, p. 478). The estimated model has the following form:

$$(18)\quad \left(\frac{P^{\theta}-1}{\theta}\right) = \beta_0 + \beta_H\left(\frac{H^{\lambda}-1}{\lambda}\right) + \beta_R R + \varepsilon$$

where P is either the price per tree or the price per foot, H is height in feet, R is a retail price dummy variable, and ε is the error term. The dummy variable ($R = 1$ if price is a retail price and $R = 0$ if price is a farm level price) is included because we merged the Fraser fir retail and farm level prices. Parameters θ and λ are transformation coefficients that, when estimated together with the other parameters, determine the appropriate relationship between price and height.[13]

The estimates of equation (18) are presented in table 3. The top panel in the table shows the estimates of the model where price per tree is a dependent variable and in the bottom panel the regression results where price per foot is a dependent variable. Using the estimated coefficients we generate the in-sample predictions of the rate of price change. The numbers presented are the averages of predictions generated for each observation. Standard errors of the rates of change are bootstrapped using Efron's procedure. Similar to the procedure used before, we first test whether the predicted rates of price change are different than the 11% interest rate. Those rates that are different are marked by an asterisk. Finally, we test whether the predicted rates of change are significantly different from each other. Both tests are performed using simple t-tests.

Proposition 1 says that as k increases, the rate of optimal per tree price change is approaching the interest rate from above. As seen from the magnitudes of the predicted rates of price changes (all in the 20–30% range), they are all significantly larger than 11% with no discernable decrease of rates across heights.

Proposition 2 says that as k increases, that the rate of optimal per foot price change is approaching the interest rate from below. The estimated per foot rates are approximately the same order of magnitude as the sample means presented in table 2 and they are significantly lower than 11% in two out of five cases. For example, the rate of increase in price per foot of a 7-foot tree over a 6-foot tree is 7.1% and it is significantly less than 11% interest rate at the 5% significance level. In addition, the implied rates of change of price per foot increase in three out of four cases. The rightmost column presents the appropriate t-statistics to use in testing whether the rates of change differ significantly between adjacent heights. For example, the rate of change in price per foot for the 8–9-foot tree is significantly larger than for the 7–8-foot tree, with the t-statistic equal to +2.6. Overall, the results of the Box–Cox model are similar to the comparison of means results obtained earlier with somewhat stronger support for proposition 2.[14]

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[13] The special cases of equation (18) include a linear function (when both θ and λ are equal to one), log–linear (when both parameters equal zero), semi-log and inverse semi-log.

[14] An additional test of the above propositions could be to simulate the theoretically correct price changes across tree cohorts based on equations (15) and (17) for the relevant range of interest rates and compare them with the empirically observed rates.

Conclusions

In this article we examined the observed age–price relationship in a competitive Christmas tree market in order to test a Hotelling–Faustmann model of natural resource pricing. The fact that all Christmas tree sellers price their trees so that every age (height) class commands a different price and that prices are posted in dollars per tree prompted a two-pronged investigation. Using a price per tree approach, the theoretical result shows that the steady-state equilibrium relationship for the rate of change in prices between two adjacent tree cohorts is identical to the Faustmann optimal rotation formula. However, when prices are calculated on a per foot basis, the equilibrium rate of change in price between two age groups turns out to be a modified Faustmann rule with the modification introduced by the percentage growth factor that is inversely related to the growth of the trees.

Using cross-sectional survey data on North Carolina Christmas tree prices from December 1997 we find that, in general, the price ratios of adjacent age cohorts conform to the modified Hotelling–Faustmann pricing rules, reflecting a competitive equilibrium in the capital market. In particular we were able to find a reasonably strong empirical evidence for two results.

First, on a per tree basis, the age structure of the stumpage prices reveal the pattern where prices across age cohorts increase at the rate higher than the interest rate. The percent increase in price above the rate of interest captures the site effect; that is, the possibility that the site can be replanted, after marketable trees are harvested. When the resource owner forgoes harvesting a Christmas tree, he not only forgoes the current revenue, he also forgoes the replanting that could be started, therefore he must be compensated for both. Strong empirical support for the diminishing importance of the site effect as the age of a tree increases is lacking, most probably being dampened by the asymmetric cost structure. The fact that the annual cost per tree of a given harvestable cohort is larger than the compounded annual per tree cost of a one year younger cohort to a certain degree offsets the decreasing importance of the site effect.

Second, our data also show that the rate of change in price expressed on a per foot basis is smaller than the interest rate and that it increases with age (height) of the tree. The theory predicts that the percentage increase in price per foot is negatively impacted by the biological growth rate, which is declining as trees grow older. Empirical support for the compensating effect of growth effect diminishing in older cohorts is somewhat weaker perhaps because of the same unmeasurable cost effects. The obtained results seem to be explaining the puzzle of why Christmas tree dealers do not price their trees on a per foot basis. The answer is simple. A foot of an older tree is more valuable than a foot on a younger tree.

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Doing this shows that the empirical data matches well with the simulated Hotelling-Faustmann price ratios. Using the 11% interest rate, it turns out that 70% of the predicted price ratios fall inside the 95% confidence intervals for the means of the data. The predictions are especially precise for shorter cohorts of the farm level prices.

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