EFFECTIVENESS OF DUAL HEDGING WITH PRICE AND YIELD FUTURES

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INTRODUCTION

Futures price contracts and options on price futures have been used for quite some time to manage price risk. However, similar market-based instruments for managing yield risk have not been available. Instead, federal agricultural support programs such as deficiency payments and nonrecourse loan programs along with subsidized crop yield insurance programs have served as alternatives to market-based risk management mechanisms. On 2 June 1995, the Chicago Board of Trade (CBOT) launched an interesting innovation into the agricultural markets, namely the quantity-based crop yield futures and options contracts. The first listed for trading was the Iowa Corn Yield Insurance Futures and Options contract, which started with fairly modest volume and open interest. Then, surprisingly, on 19 January 1996, the CBOT added a U.S. contract plus four additional state corn yield contracts for Illinois, Indiana, Ohio, and Nebraska, thereby further diluting the already thin market in the existing Iowa contract. The idea behind the move was that the low cor-

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relation between the Iowa state-average yield and individual farms' yields across the country represents a limiting factor toward more rapid acceptance of the yield contract as a revenue insurance vehicle and that adding more contracts would partially solve the problem. As it turned out, the introduction of more corn yield contracts did not increase the liquidity of the market, and the original plan to add several other yield contracts such as Illinois Soybean, Kansas Winter Wheat, and North Dakota Spring Wheat has been scrapped. As of May 1997, the six corn yield contracts are still officially listed for trading at the CBOT, but the exchange records hardly any action at all.

This article is motivated by several interesting empirical problems surrounding the issue of the success/failure of the newly introduced yield futures contract. Casting the problem in the risk minimization framework and thereby measuring the hedging effectiveness by the reduction in the variance of revenue, two questions emerge. What are the factors determining the hedging effectiveness of yield futures? Are those factors strong enough to guarantee revenue stabilization to producers in regions outside the geographical coverage of the yield contract? The effectiveness of the dual hedging strategy with price and yield futures is examined using the example of three hypothetical corn producers located in the major corn producing counties in North Carolina. The idea is that corn yields in North Carolina are sufficiently different from corn yields in Iowa to make the answers to the questions meaningful.

The behavior of a firm under joint price and output uncertainty has been studied previously in the literature. In a pioneering work, McKinnon (1967) showed that, since the correlation between individual yield and local price is typically negative, the risk minimizing hedge against price risk is less than the expected output. Extensions of McKinnon's risk minimization analysis have generally focused on attempts to cast the problem into the expected utility maximization framework. Since the general solution to the expected utility maximization problem is not analytically tractable, most authors have embraced some assumptions regarding the form of the utility function and the distribution of the random variables. Standard examples include the joint normality of profits and prices (Grant, 1985), or constant absolute risk aversion (CARA) and normally distributed profits/revenues which reduce the expected utility maximization approach to the analytically simpler mean-variance framework (Rolfo, 1980). Problems with assuming the normality of profits, which is the product of two random variables (price and quantity), were addressed in Lapan and Moschini (1994). They derived an exact solution for the optimal futures hedge for the CARA utility function and jointly, normally distributed price, basis, and yield, but not profits. Vukina, Li, and Holthausen (1996) have shown that even with two instruments (price and yield futures contracts) for managing the price and yield risks and no price basis and yield basis risks, the firm still cannot generally eliminate all uncertainty. However, even in the presence of both price and yield basis risks, a risk minimizing firm can reduce its variance of revenue by hedging in both markets rather than just using the price futures market.

The theoretical results show that the effectiveness of the dual hedge depends, critically, on the price and yield basis risks and that the introduction of the second instrument into the risk minimizing hedging strategy generally increases the demand for the first instrument. The empirical results indicate a small, although statistically significant, reduction in the variance of revenue caused by hedging in both markets rather than just using the price futures market for two of the three analyzed counties.

THE MODEL

The traditional role of hedging is risk reduction. With the presence of joint price and output uncertainties and the availability of two hedging instruments, the problem facing the agricultural firm is one of selecting the optimal position in both price and yield futures markets that will minimize the variance of revenue. Consider a two-period problem. In the first (planting) period, all decisions regarding futures positions are made, and in the second (harvesting) period, all uncertainties are resolved and all outstanding positions are closed and proceeds collected. Of course, minimizing the variance of revenue makes sense only for a given scale of production, because otherwise a farmer could always minimize risk by producing nothing. Therefore, the scale of production (the number of acres planted), X, is exogenous to the farmer's decision problem, and the total output is Xy, where y denotes stochastic yield. The problem can be defined as

$$\min_{h_f, h_z} E[M - E(M)]^2$$

$$s.t.$$

$$M = pXy + (F - f)h_f + K(Z - z)h_z \tag{1}$$

where M represents joint income from cash marketing and futures trading, p is local cash price, F is futures price at planting, f is futures price at harvest, h_f is price futures market position (positive if short, negative if long), y is individual farm's yield (bushels/acre), Z is contract underlying yield at planting, z is contract underlying yield at harvest, and h_z is yield futures market position (positive if short, negative if long). The underlying instrument of the Iowa Corn Yield Insurance Futures is the official state-based yield estimates released during the growing and harvesting season by the U.S. Department of Agriculture (USDA). The unit of trading is the Iowa yield estimate in bushels/acre harvested times a multiplier, $K =$ \$100 (e.g., a yield of 132.2 bu./acre gives a contract value of \$13,220). Contract months are September and January, and contracts are settled in cash following the release of the USDA's report. The lowercase symbols, p, f, y, and z, denote stochastic variables, and the uppercase letters, F, Z, X, and K, denote known constants. The formulation of the problem in eq. (1) captures the presence of both price and yield basis risks.

Assuming that the random variables have finite and time invariant second moments, the minimization problem in eq. (1) translates into

$$\min_{h_f, h_z} \operatorname{Var}(M) = \sigma_R^2 + h_f^2 \sigma_f^2 + K^2 h_z^2 \sigma_z^2 - 2h_f \sigma_{Rf} - 2Kh_z \sigma_{Rz} + 2Kh_f h_z \sigma_{fz} \tag{1'}$$

where $\sigma_R^2 = \operatorname{Var}(R) = \operatorname{Var}(pXy)$ is the variance of revenue without hedging (cash marketing), $\sigma_f^2 = \operatorname{var}(f)$, $\sigma_z^2 = \operatorname{var}(z)$, $\sigma_{Rf} = \operatorname{cov}(R, f)$, $\sigma_{fz} = \operatorname{cov}(z, f)$, and $\sigma_{Rz} = \operatorname{cov}(R, z)$. Solving eq. (1′) gives the formulae for the risk minimizing hedges in price and yield futures:

$$\hat{h}_f = \frac{\sigma_R}{\sigma_f(1 - \rho_{fz}^2)} (\rho_{Rf} - \rho_{fz}\rho_{Rz}) \tag{2.1}$$

$$\hat{h}_z = \frac{\sigma_R}{K\sigma_z(1 - \rho_{fz}^2)} (\rho_{Rz} - \rho_{fz}\rho_{Rf}) \tag{2.2}$$

with ρ denoting correlation coefficients. For instance, $\rho_{fz} = \sigma_{fz}/\sigma_f\sigma_z$ is the correlation coefficient between futures price and contract underlying yield.[1] It is assumed that the Hessian of Var(M) is a positive definite and symmetric matrix such that the first order conditions describe strict local minimum. Consequently, the denominators of eqs. (2.1) and (2.2) are positive, and both price and yield hedges can be either short or long depending on the signs and relative magnitudes of the correlation terms.

The above results are not based on any assumption about the distribution of random variables. However, further insights into the problem can be developed by decomposing correlation coefficients involving the

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[1] By replacing correlation coefficients with corresponding variances and covariances, the results (2.1) and (2.2) become identical to those obtained by Vukina et al. (1996).

revenue part (R) in terms of their fundamental random variables, p and y. To accomplish this, one has to assume that the stochastic variables, p, f, y, and z, are multivariate normal.[2] Following this assumption, Vukina et al. (1996) show that

$$\rho_{Rf} = \frac{X}{\sigma_R} (\mu_y \sigma_p \rho_{pf} + \mu_p \sigma_y \rho_{yf}) \tag{3.1}$$

$$\rho_{Rz} = \frac{X}{\sigma_R} (\mu_y \sigma_p \rho_{pz} + \mu_p \sigma_y \rho_{yz}) \tag{3.2}$$

where $\mu_y = E(y)$ and $\mu_p = E(p)$. Equation (3.1) indicates that the correlation coefficient between revenue and futures price can be expressed as weighted sum of the correlation coefficient between cash and futures price (ρ_{pf}) and the correlation coefficient between the farm's yield and futures price (ρ_{yf}) because the third moment of ρ_{Rf} disappears under the joint normality of p, y, and f. Substituting eqs. (3.1) and (3.2) into eqs. (2.1) and (2.2) yields risk minimizing dual hedges under normality:

$$\hat{h}_f = \frac{X}{\sigma_f(1 - \rho_{fz}^2)} [\mu_y \sigma_p (\rho_{pf} - \rho_{pz}\rho_{fz}) + \mu_p \sigma_y (\rho_{yf} - \rho_{fz}\rho_{yz})] \tag{4.1}$$

$$\hat{h}_z = \frac{X}{K\sigma_z(1 - \rho_{fz}^2)} [\mu_p \sigma_y (\rho_{yz} - \rho_{fz}\rho_{yf}) + \mu_y \sigma_p (\rho_{pz} - \rho_{fz}\rho_{pf})] \tag{4.2}$$

The interpretation of the hedging demand functions (4.1) and (4.2) is facilitated by noticing that one minus the correlation coefficient between cash price and futures price ($1 - \rho_{pf}$) represents a natural measure of the price basis risk, whereas ($1 - \rho_{yz}$) can be used as a measure of yield basis risk. As indicated by eq. (4.2), the risk minimizing demand for yield contracts is inversely related to the yield basis risk [as ρ_{yz} increases, the yield basis risk ($1 - \rho_{yz}$) decreases] and the volatility of the contract underlying yield (σ_z).

The first part of the expression (4.2), namely, $X\mu_p\sigma_y(\rho_{yz} - \rho_{fz}\rho_{yf})/K\sigma_z(1 - \rho_{fz}^2)$, represents the "direct hedging" demand for yield futures, in the sense that it minimizes yield risk holding cash price constant. If, in addition to holding the cash price constant, it is assumed that the futures price is constant as well, the direct hedging position reduces to a simple risk minimizing hedge in yield futures, $X\mu_p\sigma_y\rho_{yz}/K\sigma_z$. This expression can be referred to as the "primary direct hedge" because this portion

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[2] As correctly pointed out by an anonymous referee, the assumption of normally distributed yields is not supported empirically. However, the insights gained by assuming joint normality probably justify the use of this admittedly restrictive assumption.

of the yield contracts demanded serves the sole purpose of minimizing yield risk, assuming there are no price risk and price futures contracts. The leftover part of the direct hedging component is the adjustment to the primary direct hedge when the constant futures price assumption is relaxed. The adjustment term, given by the product between ρ_{fz} and ρ_{yf}, reflects the effect of the interaction between futures price with cash and futures yield on the demand for yield contract.

The second part of the expression (4.2), namely, $X\mu_y\sigma_p(\rho_{pz} - \rho_{fz}\rho_{pf})/K\sigma_t(1 - \rho_{fz}^2)$, is the "cross-hedging" demand for yield contracts in the sense that this portion of the demand for yield futures is used to hedge price risk holding the spot yield constant. Again, the first component of the cross-hedge part is the primary cross-hedge and the second term reflects the influence of price futures on the cross-hedging demand for yield contracts. Similar interpretation holds for the risk minimizing hedge in price futures given by eq. (4.1).

HEDGING EFFECTIVENESS OF YIELD CONTRACTS

In the risk minimizing framework, hedging effectiveness is measured by the reduction in the variance of revenues. One can compare the reduction in the variance of revenue between various hedging strategies and the cash marketing strategy, or between two different hedging strategies. Using $R = pXy$ for the revenue without hedging, one can define $R_{f0} = R + (F - f)\hat{h}_{f0}$ as the revenue resulting from hedging using only the risk minimizing hedge in price futures, $R_{0z} = R + K(Z - z)\hat{h}_{0z}$ as the revenue resulting from hedging using only the risk minimizing yield hedge, and $R_{fz} = R + (F - f)\hat{h}_f + K(Z - z)\hat{h}_z$ as the revenue resulting from hedging using both price and yield futures. The following four cases under joint price and yield risk are investigated:

$$\Delta_1 = \mathrm{Var}(R) - \mathrm{Var}(R_{f0}) = \sigma_f^2(\hat{h}_{f0})^2 \geq 0 \tag{5.1}$$

$$\Delta_2 = \mathrm{Var}(R) - \mathrm{Var}(R_{0z}) = K^2\sigma_z^2(\hat{h}_{0z})^2 \geq 0 \tag{5.2}$$

$$\Delta_3 = \mathrm{Var}(R_{f0}) - \mathrm{Var}(R_{fz}) = K^2\sigma_z^2(1 - \rho_{fz}^2)(\hat{h}_z)^2 \geq 0 \tag{5.3}$$

$$\Delta_4 = \mathrm{Var}(R) - \mathrm{Var}(R_{fz}) = \Delta_3 + \frac{\sigma_{Rf}^2}{\sigma_f^2} \geq 0 \tag{5.4}$$

The expression (5.1) is the variance of revenue eliminated by single hedging in price futures compared to the cash marketing only strategy, and $\hat{h}_{f0}$ is the risk minimizing hedge in price futures under joint price and yield risk, assuming the yield futures contract does not exist. The expression (5.2) is the variance of revenue eliminated by single hedging in yield futures compared to the cash marketing strategy, and $\hat{h}_{0z}$ is the risk minimizing hedge in yield futures, assuming a price futures contract does not exist.[3] Expression (5.3) represents the reduction in revenue variance by entering the crop yield futures market in excess of being hedged in the price futures, and expression (5.4) represents the reduction in the variance of revenue from the dual hedging over the cash marketing strategy.[4]

Expressions (5.1) and (5.2) are always nonnegative, indicating that engaging in hedging with either price futures or yield futures contracts has the potential to reduce the variance of revenue over cash marketing strategy. Simultaneous hedging in price and yield futures enables further reduction in the variance of revenue. Expression (5.3) is always nonnegative regardless of whether the risk minimizing hedge in yield futures is long or short. If Δ_3 equals zero, hedging in crop yield futures does not contribute to the variance reduction. By construction, Δ_3 equals zero only if the risk minimizing yield futures position equals zero. Notice that the above conclusions are free of assumptions about the distribution of random variables. The expressions for Δ_1, Δ_2, and Δ_3 can be evaluated by using either the general formulae for risk minimizing hedges or those based on joint normality. For example, the sign of eq. (5.3) is invariant to whether $\hat{h}_z$ is expressed by eq. (2.2) or by its normal counterpart (4.2).

However, to figure out which factors influence the hedging effectiveness of these hedging strategies and in what direction, one has to make an assumption about the joint normality of prices and yields. For example, referring to the formula for the optimal single hedge in yield futures under normality in footnote 3, one can see that the reduction in the variance of revenue generated by hedging in yield futures compared to the cash marketing strategy (Δ_2) is determined by the weighted sum of the correlation coefficient between cash price and futures yield ($\rho_{pz} <$

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[3] The formulae for the single risk minimizing hedges, $\hat{h}_{f0}$ and $\hat{h}_{0z}$, are given in eqs. (A1.1) and (A1.2). Introducing normality, i.e., using expressions (3.1) and (3.2), eqs. (A1.1) and (A1.2) become

$$\hat{h}_{f0} = \frac{X}{\sigma_f}(\mu_y\sigma_p\rho_{pf} + \mu_p\sigma_y\rho_{yf}) \text{ and } \hat{h}_{0z} = \frac{X}{K\sigma_z}(\mu_y\sigma_p\rho_{pz} + \mu_p\sigma_y\rho_{yz})$$

[4] The results for Δ_4 in eq. (5.4) follows directly from the definition of Δ_3 and the result for

$$\mathrm{Var}(R_{f0}) = \sigma_R^2 - \frac{\sigma_{Rf}^2}{\sigma_f^2}.$$

0) and the correlation coefficient between the farm's yield and futures yield ($\rho_{yz} > 0$). Differentiating Δ_2 with respect to ρ_{yz} explains the impact of the yield basis on hedging effectiveness:

$$\frac{\partial \Delta_2}{\partial \rho_{yz}} = 2KX\sigma_y\sigma_z\mu_p(\hat{h}_{0z}) \tag{6}$$

The sign of the derivative is positive only if the risk minimizing hedge is short. Hence, a decrease in the yield basis risk, i.e., an increase in the correlation between spot yield and futures yield (ρ_{yz}), enhances the hedging performance of the risk minimizing short hedge in yield futures contracts compared to the cash marketing strategy. Contrary to that, if the risk minimizing hedge is long, an increase in the yield basis risk improves hedging effectiveness. Parallel comparative statics result is obtained for the reduction in revenue variance caused by single hedging in price futures contracts over the cash marketing only strategy (Δ_1).

As far as the hedging effectiveness of the dual hedging strategy is concerned, the comparative statics results are quite similar. As seen from eq. (5.3), the reduction of revenue variance by entering the crop yield futures market in addition to hedging with price futures depends on the sign of the risk minimizing yield hedge. If the optimal yield hedge is short, an increase in price (yield) basis risk, i.e., a decrease in correlation between cash and futures price (spot yield and futures yield), reduces the hedging effectiveness of the dual hedging over the price-futures-only hedge. Contrary to that, if the optimal yield hedge is long (an empirically unlikely scenario), an increase in price (yield) basis risk improves hedging effectiveness of the two-instrument hedging strategy over the price-futures-only strategy (for details see Vukina et al., 1996).

Before the introduction of the yield futures contract, only price futures contracts could have been used to hedge both price and yield risks. Researchers have noticed that, allowing for yield uncertainty, the negative correlation between yield and futures price ($\rho_{yf} < 0$) causes the optimal price futures hedge to become smaller compared to the case of nonrandom yields. The negative correlation between yield and futures price partially offsets the hedging demand for price futures. This result has become known in the literature as the "natural hedge" (McKinnon, 1967; Lapan and Moschini, 1994).

The introduction of the yield futures as a second instrument into the risk minimizing hedging strategy changes this result in the sense that, under certain conditions, instruments become complementary: i.e., the introduction of the second instrument increases the demand for the first instrument.

Proposition

Assuming stochastic prices and yields, the price (yield) futures hedge in a two-instrument strategy is always larger than the price (yield) futures hedge in a one-instrument strategy, provided that both one-instrument positions are short.

Proof

Inserting the risk minimizing single hedges into the price and yield futures demands from the risk minimizing dual strategy (see Appendix A) yields:

$$\hat{h}_f - \hat{h}_{f0} = \frac{\sigma_R \rho_{Rf} \rho_{fz}^2}{\sigma_f (1 - \rho_{fz}^2)} - \frac{\sigma_R \rho_{Rz} \rho_{fz}}{\sigma_f (1 - \rho_{fz}^2)} \tag{7.1}$$

$$\hat{h}_z - \hat{h}_{0z} = \frac{\sigma_R \rho_{Rz} \rho_{fz}^2}{K\sigma_z (1 - \rho_{fz}^2)} - \frac{\sigma_R \rho_{Rf} \rho_{fz}}{K\sigma_z (1 - \rho_{fz}^2)} \tag{7.2}$$

Since futures price and futures yield are negatively correlated ($\rho_{fz} < 0$), it is easily shown that $\hat{h}_f > \hat{h}_{f0}$ and $\hat{h}_z > \hat{h}_{0z}$ if $\rho_{Rf} > 0$ and $\rho_{Rz} > 0$, which are exactly the conditions for the single hedge positions, $\hat{h}_{f0}$ and $\hat{h}_{0z}$, being short (positive). The exact opposite is true if single instrument hedges are both long (negative). The results are ambiguous when optimal single instrument hedges have opposite signs. This conclusion is valid for any distribution of stochastic variables, p, f, y, and z, with finite variance-covariance matrix.

Q.E.D.

The fact that the addition of the yield futures increases the demand for the price futures when the risk minimizing strategy calls for the establishment of a short hedge (which is, empirically, generally the case) suggests that price and yield contracts are complements. The complementary effect of price and yield futures depends on the correlation between price and yield futures. One extreme is the case where $\rho_{fz} = 0$. A zero correlation coefficient between futures price and futures yield will totally eliminate the complementary effect, i.e., $\hat{h}_z = \hat{h}_{0z}$ and $\hat{h}_f = \hat{h}_{f0}$. An increase in the correlation between futures price and futures yield tends to raise the demand for both price and yield contracts and, therefore, strengthens the complementary effect. The futures price of a commodity is the market anticipation of the aggregate quantity of goods demanded and supplied. The futures yield reflects the state-average yield forecast, e.g., the Iowa average corn yield in the case of Iowa corn yield futures. To raise the correlation between the price and yield futures, one would have to enlarge the geographic coverage of the yield futures contract, e.g., from a state-average yield to a regional- or national-average yield. On the other hand, increasing the geographic coverage of the yield futures contract tends to make the market quote of yield futures less relevant to individual farmers. Consequently, this may decrease the benefit from using the yield contract for hedging purposes. These opposite effects of the correlation between futures price and futures yield on the hedging demand for both price and yield futures contracts have important implications for the design of optimal yield contracts.

Finally, the complementarity between contracts depends on the volatility of futures price and futures yield. As shown from eq. (7.1), an increase in the volatility of futures price, as measured by σ_f, decreases the difference between $\hat{h}_f$ and $\hat{h}_{f0}$, causing a reduction in demand for the price contract as a result of adding the yield contract. Similarly, as shown from eq. (7.2), an increase in the volatility of futures yield (σ_z) decreases the demand for the yield contract caused by addition of the price contract.

AN EXAMPLE: NORTH CAROLINA CORN PRODUCERS

To calculate the theoretical reduction in the variance of revenue under three different marketing scenarios from the preceding section, the three leading corn producing counties in North Carolina are selected. One local spot price in each county is used for county level estimates: Elizabeth City for Pasquotank County, Greenville for Pitt County, and Lumberton for Robeson County. Since most of the corn in North Carolina is harvested in the period between September 1 and October 15, the annual observations used are the averages of weekly observations (Thursdays) within this six-week period. The cash prices are No. 2 yellow shelled corn prices paid to producers for grain delivered in bulk to elevators and are taken from various issues of *Weekly Grain Report* of the North Carolina Department of Agriculture. The futures prices are settlement prices for the CBOT December corn futures. Farm level yields are approximated with the average county yields. Since yield contracts started trading relatively recently, historical series of trading data on yield futures do not exist, and hence are approximated with the realized Iowa state-average yield. The scale of production is fixed at 500 acres.

Allowing for joint price and yield uncertainty, the empirical analysis is performed for three different risk minimizing hedge scenarios: (i) hedging with price futures only using formula (A1.1); (ii) hedging with yield futures only using formula (A1.2); and (iii) simultaneous hedging with both price and yield futures using formulae (2.1) and (2.2). To compute the risk minimizing hedges, it is necessary to quantify the individual decision maker's expectations about the realization of random variables and the perception of risk. The approach implemented in this study uses price forecasts to approximate subjective price expectations and the variance-covariance matrix of forecast errors to measure risk. It is assumed that the decision maker's information set includes the historical prices and yields in the harvesting period of each year. The available data set used in this study covers the 1951–1994 period, for a total of 44 annual observations.

Estimation and forecasting involve two steps. To correct for possible deterministic nonstationarities, all time series are first detrended by fitting the linear time trend through the nominal data. In the case of prices, the estimated time trend takes care of inflation, whereas in the case of yields, it corrects for the technological change that might have occurred in corn production during that period. Additionally, in the case of prices, the time trend is augmented by the dummy variable for the period 1973–1994 to account for the dramatic changes in the behavior of prices after 1973 (caused by the combination of various factors such as the world oil crisis and the collapse of the fixed exchange rates system, to name only the two most important ones). The ordinary least squares (OLS) estimates of the individual linear models are summarized in Table I.

In the second step, two models are experimented with. First, the detrended series is modeled assuming no restrictions on the distribution of random variables, which leads to a multivariate time series forecasting model with three random variables (cash revenue, futures price, and futures yield) jointly modeled. Second, the joint normality of prices and yields is assumed, which leads to a vector-valued time series forecasting model with four random variables (cash price, spot yield, futures price, and futures yield) jointly modeled. In the latter approach, expressions (3.1) and (3.2) are used to decompose the correlation coefficient between cash revenue and futures price (ρ_{Rf}) and the correlation coefficient between cash revenue and futures yield (ρ_{Rz}) in terms of the fundamental variables, p and y. The detrended series are modeled as vector-valued weakly stationary processes using Aoki's (1987) state-space forecasting procedure. The description of the state-space parameters estimation and forecasting procedures of minimal complexity appears in Vukina (1992). The estimation results presented in the form of correlation coefficients of in-sample forecast errors for three-variable and four-variable models are presented in Tables IIA and IIB.

TABLE I

Time Trend Estimation Results

Model	*Pasquotank County*	*Pitt County*	*Robeson County*
Dependent variable: cash revenue (R)			
Constant	41,950.0[a]	31,611.0[a]	24,395.0[a]
T	135.71	176.99	388.5
$D_{73\text{–}94}$	77,733.0[a]	50,582.0[a]	53,466.0[a]
Adjusted R^2	0.7923	0.6550	0.6915
Dependent variable: futures price (f)			
Constant		1.5663[a]	
T		−0.025532[a]	
$D_{73\text{–}94}$		1.8548[a]	
Adjusted R^2		0.7576	
Dependent variable: futures yield (z)			
Constant		52.29[a]	
T		1.821[a]	
Adjusted R^2		0.7093	
Dependent variable: cash price (p)			
Constant	1.5454[a]	1.482[a]	1.5716[a]
T	−0.019067[b]	−0.016606	−0.018584[b]
$D_{73\text{–}94}$	1.534[a]	1.516[a]	1.4896[a]
Adjusted R^2	0.6859	0.6942	0.6765
Dependent variable: spot yield (y)			
Constant	50.225[a]	42.518[a]	29.652[a]
T	1.5324[a]	0.91839[a]	1.3326[a]
Adjusted R^2	0.6517	0.3117	0.4973

[a] Significant at the 1% level.
[b] Significant at the 5% level.

The in-sample predictive power of the models is measured by the mean absolute percentage error (MAPE) statistics. The four-variable models generally outperform the three-variable models, largely due to the relatively poor forecasting results for the revenue variable. As seen from Tables IIA and IIB, the MAPE for R variable in Robeson county amounts to 29% and in Pitt county 26%. Among the four-variable models, the worst forecasting results are recorded for the spot yields in Pitt (25%) and Robeson (26%) counties. For the remaining variables, the MAPE statistics are generally within the 11–13% range.

The information from Tables IIA and IIB in the form of the forecast error variance-covariance matrices is used to calculate risk minimizing hedging positions as well as the reductions in the variance of revenue due to various hedging strategies. These results are presented in Tables IIIA and IIIB. The third and fourth rows in the tables give the percentage reduction in the variance of revenue due to implementation of various

TABLE IIA

Estimated Correlation Coefficient Matrix of Forecast Errors: Three Random Variables (R, f, z) Jointly Modeled

	Cash Revenue (R)	*Futures Price (f)*	*Futures Yield (z)*
	Pasquotank County		
MAPE[a]	18.17%	11.75%	12.21%
Cash revenue (R)	1	0.64	−0.03
Futures price (f)		1	−0.18
Futures yield (z)			1
	Pitt County		
MAPE	25.53%	12.34%	12.49%
Cash revenue (R)	1	0.54	0.20
Futures price (f)		1	−0.29
Futures yield (z)			1
	Robeson County		
MAPE	29.06%	13.03%	12.01%
Cash revenue (R)	1	0.57	0.11
Futures price (f)		1	−0.27
Futures yield (z)			1

[a] MAPE = mean absolute percentage error.

hedging strategies. The basis for comparison in the third row is the variance of cash (no hedge) revenue [$\sigma_R^2 = \mathrm{Var}(R)$] and the basis for comparison in the fourth row is the variance of the price-hedge-only revenue (R_{f0}). In either situation, the estimate of σ_R^2 is needed. In the three-series models where cash revenue is modeled jointly with futures price and futures yield, this measurement can be recovered directly from the model as the forecast error variance of revenue. Using the four-series model where the fundamental variables, p and y, are modeled rather then their product, an analytical expression for σ_R^2 is needed. Under the assumption of joint normality between p and y, $\mathrm{Var}(R)$ can be decomposed into a linear function of variances of p and y and covariance between p and y (see Appendix B).

The results for all three counties are fairly similar. The joint normality of random variables assumption case (Table IIIB) typically produces larger reductions in the variance of revenue than the no distributional assumption case (Table IIIA). The correlation coefficients between cash price and futures price are relatively high for all three counties: 0.95 for Pasquotank, 0.92 for Pitt, and 0.91 for Robeson. In the no distributional assumption case, the introduction of hedging in price futures reduces the variance of cash marketing revenue by 41% in Pasquotank

TABLE IIB

Estimated Correlation Coefficient Matrix of Forecast Errors: Four Random Variables (p, y, f, z) Jointly Modeled

	Cash Price (p)	*Spot Yield* (y)	*Futures Price* (f)	*Futures Yield* (z)
		Pasquotank County:		
MAPE[a]	13.19%	15.51%	13.25%	11.71%
Cash price (p)	1	−0.07	0.95	−0.20
Spot yield (y)		1	0	0.16
Futures price (f)			1	−0.22
Futures yield (z)				1
		Pitt County		
MAPE	12.77%	24.95%	12.99%	12.01%
Cash price (p)	1	−0.04	0.92	−0.27
Spot yield (y)		1	0.10	0.46
Futures price (f)			1	−0.24
Futures yield (z)				1
		Robeson County		
MAPE	11.84%	26.18%	11.44%	11.74%
Cash price (p)	1	−0.19	0.91	−0.33
Spot yield (y)		1	0.02	0.44
Futures price (f)			1	−0.28
Futures yield (z)				1

[a] MAPE = mean absolute percentage error.

county, 29% in Pitt county, and 32% in Robeson county. The numbers for the normality assumption case are 66% for Pasquotank, 46% for Pitt, and 41% for Robeson. The correlation coefficients between county level yield and Iowa state-average yield are as low as 0.16 for Pasquotank county and as high as 0.46 for Pitt county. Using only yield contracts to hedge against price and yield risks generates substantially smaller reductions in the variance of revenue than using only price contracts. In the three-series model case, these reductions amount to 0.12% for Pasquotank, 4% for Pitt, and 1.2% for Robeson counties and are achieved by going long 0.43 contracts of yield futures in Pasquotank county, going short 2.6 contracts in Pitt county, and going short 1.4 contracts in Robeson county.

Using price futures contracts in combination with yield futures contracts generates more pronounced variance of revenue reductions than using the price contracts alone. In the four-series model case, further reduction in the variance of revenue generated by the inclusion of the yield futures hedging instrument over the price hedge strategy amounts to 2.84% in Pasquotank, 23.87% in Pitt, and 19.02% in Robeson. Relatively smaller reduction in the variance of revenue generated by the dual

TABLE IIIA

Comparison of Various Hedging Scenarios for 500 Acres of Corn: Three Random Variables Jointly Modeled (No Distributional Assumptions)

	Price Futures Only	*Yield Futures Only*	*Price and Yield Futures*
	Pasquotank County		
No. of price contracts	7.56		7.75
No. of yield contracts		−0.43	1.05
Variance reduction over no hedge	41.26%	0.12%	41.94%
Variance reduction over price hedge			1.15%
	Pitt County		
No. of price contracts	5.79		6.99
No. of yield contracts		2.56	4.96
Variance reduction over no hedge	28.96%	3.99%	42.68%
Variance reduction over price hedge			19.31%
	Robeson County		
No. of price contracts	6.09		6.91
No. of yield contracts		1.40	3.61
Variance reduction over no hedge	32.25%	1.20%	39.64%
Variance reduction over price hedge			10.90%

TABLE IIIB

Comparison of Various Hedging Scenarios for 500 Acres of Corn: Four Random Variables Jointly Modeled (Assuming Joint Normality)

	Price Futures Only	*Yield Futures Only*	*Price and Yield Futures*
	Pasquotank County		
No. of price contracts	11.14		11.45
No. of yield contracts		−1.38	1.72
Variance reduction over no hedge	65.77%	0.66%	66.74%
Variance reduction over price hedge			2.84%
	Pitt County		
No. of price contracts	8.48		9.57
No. of yield contracts		2.69	5.36
Variance reduction over no hedge	46.33%	3.44%	59.14%
Variance reduction over price hedge			23.87%
	Robeson County		
No. of price contracts	8.69		10.02
No. of yield contracts		2.03	5.05
Variance reduction over no hedge	41.26%	1.96%	52.43%
Variance reduction over price hedge			19.02%

hedging strategy over the single price futures hedging strategy in Pasquotank county can be explained by the low correlation between county level yields and the Iowa state average. Stronger correlation between county level yield and the contract underlying yield (such as the case in Pitt county -0.46) produces a larger additional variance reduction (23.9%).

Tables IIIA and IIIB also show the complementary effect of the price and yield contracts. For all three counties, with and without distributional assumption, the hedging positions from dual hedging strategies exceed their counterparts from single hedge strategies. In general, yield hedges exhibit much stronger complementary effects than price hedges. In Pitt county, for instance, under normality assumption, the optimal price hedge increased from 8.48 in the single contract strategy to 9.57 contracts in the dual strategy, whereas the optimal yield hedge increased from 2.69 to 5.36 contracts.

At this stage, the natural question to ask is whether the calculated reductions in the variability of revenue between various scenarios are statistically different from each other. To answer that question, the 95% bootstrapped confidence intervals for each percentage reduction in the variance of revenue is constructed and the results are presented in Tables IVA and IVB. Bootstrapping is chosen because the analytical formula for standard errors of the required statistics may be very complicated, or even impossible, to derive. The procedure used involves four steps:

1. Checking the whiteness of residuals from the state-space multivariate time series models. If the model is properly specified the residuals should be white noise. Using the Fisher-Kappa test (Fuller, 1976), the results do not reject the null hypotheses that the forecasting errors are white noise in 19 of 21 times series. The only two cases where the white noise hypothesis is rejected are local yield residuals in Pitt and Robeson counties in the four time series (p, f, y, z) models.
2. Drawing 10,000 independent bootstrap samples with replacement from the white noise forecasting errors, each sample consists of 44 observations. The sampling is done by picking three (or four, when assuming the joint normality of the stochastic variables) contemporaneous errors at once to maintain their contemporaneous covariance structure.
3. Computing the bootstrap replication corresponding to each bootstrap sample. In this case, a bootstrap replication is the percentage variance reduction from different hedging scenarios (Δ_1, Δ_2, Δ_3, Δ_4).
4. Since some of the histograms of 10,000 bootstrap replications exhibit asymmetrical distributions, the bias-corrected and accelerated (BCa)

TABLE IVA

Bootstrapped 95% Confidence Intervals of the Percentage Variance Reduction over No-Hedge (Cash-Only) Strategy

County	*Distributional Assumption*	*Estimate of Variance Reduction (%)*	*Lower Confidence Limit (%)*	*Upper Confidence Limit (%)*
		Single Hedge: Price Futures		
Pasquotank	None	41.26	12.64	64.78
	Normal	65.77	46.25	80.06
Pitt	None	28.96	7.20	54.35
	Normal	46.33	25.41	62.91
Robeson	None	32.25	10.26	54.37
	Normal	41.26	17.88	59.75
		Single Hedge: Yield Futures		
Pasquotank	None	0.12	0	0.38
	Normal	0.66	0	6.38
Pitt	None	3.99	0	28.67
	Normal	3.44	0	25.75
Robeson	None	1.20	0	54.37
	Normal	1.96	0	17.01

TABLE IVB

Bootstrapped 95% Confidence Intervals of Variance Reduction Due to Dual Hedging

County	*Distributional Assumption*	*Estimate of Variance Reduction (%)*	*Lower Confidence Limit (%)*	*Upper Confidence Limit (%)*
		Compared to No Hedge (Cash)		
Pasquotank	None	41.94	11.80	64.55
	Normal	66.74	45.95	79.67
Pitt	None	42.68	12.92	66.67
	Normal	59.14	36.70	74.36
Robeson	None	39.64	14.62	57.59
	Normal	52.43	29.54	69.24
		Compared to Price-Futures-Only Hedge		
Pasquotank	None	1.15	0	10.74
	Normal	2.84	0	19.99
Pitt	None	19.31	0.49	58.53
	Normal	23.87	2.40	58.79
Robeson	None	10.90	0.09	48.35
	Normal	19.02	0.87	59.33

method by Efron and Tibshirani (1993) is used in constructing 95% confidence intervals.

In hedging by price futures only (Table IVA), the variance reduction under normal assumption is still larger than that without any distributional assumption. For example, the result obtained under normality for Pasquotank county shows that 95% of the time the reduction in the variance of revenue falls between 46% and 80%. The results strongly support the theoretical results obtained earlier that engaging in hedging with price futures contracts in the presence of joint price and yield uncertainty guarantees the reduction in the variance of revenue over cash marketing. On the other hand, hedging price and yield risk with yield futures only does not always result in a reduction of variance of revenue over cash marketing because the lower limits of the 95% confidence interval for all the three counties equal zero.

The two-instrument hedge outperforms the single price futures hedge for Pitt and Robeson counties. The confidence intervals are fairly large, especially for the general case and somewhat smaller for the normal case. In Pitt county, for instance, they range from 0.5% to 58.5% in the general case and from 2.4% to 58.8% in the normal case. There is no statistically significant difference in the hedging performance between hedging with single price contract and dual hedging with price and yield contracts for Pasquotank county (Table IVB). This is caused by the presence of high yield basis risk, or equivalently, by the low correlation between the Pasquotank county-average corn yield and the Iowa state-average yield.

CONCLUSIONS

This article explores the effectiveness of hedging in yield futures. Measuring the hedging effectiveness by the reduction in the variance of revenue, it is found that the effectiveness of the yield hedge depends, critically, on the basis risks and that the direction of the effect depends on the established futures position. If the risk minimizing hedge is short, a decrease in the yield basis risk, i.e., an increase in the correlation between spot yield and futures yield (ρ_{yz}), enhances the hedging performance of the risk minimizing short hedge in yield futures compared to the cash marketing strategy. Contrary to that, if the risk minimizing hedge is long, a decrease in the yield basis risk diminishes the hedging effectiveness.

As an extension of the "natural hedge" result, it is shown that the introduction of the second instrument into the risk minimizing hedging strategy generally increases the demand for the first instrument. An increase in the correlation between futures price and futures yield tends to raise the demand for both price and yield contracts and, therefore, strengthens the complementary effect.

Empirically, this article examines the effectiveness of the dual hedging strategy with price and yield futures for hypothetical corn producers located in the three major corn producing counties in North Carolina. All theoretical findings are empirically verified. The results show a small, although statistically significant, reduction in the variance of revenue caused by hedging in both markets rather than just using the price futures market for two of the three analyzed counties. Relatively modest reduction in the variance of revenue generated by the dual hedging strategy over the single price futures hedging strategy can be explained by the low correlation between individual county level yields in North Carolina and the contract underlying yield (Iowa state average).

APPENDIX A

Assuming the joint presence of price and yield risks and availability of only price futures contracts, the dual optimal price hedge from eq. (2.1) reduces to the single risk minimizing hedge in price futures:

$$\hat{h}_{f0} = \frac{\sigma_R}{\sigma_f} (\rho_{Rf}) \tag{A1.1}$$

Assuming the availability of only yield futures contracts, the dual optimal yield hedge from eq. (2.2) reduces to the single risk minimizing hedge in yield futures:

$$\hat{h}_{0z} = \frac{\sigma_R}{K\sigma_z} (\rho_{Rz}) \tag{A1.2}$$

Both $\hat{h}_{f0}$ and $\hat{h}_{0z}$ can be either long $(-)$ or short $(+)$ since ρ_{Rf} and ρ_{Rz} may be positive or negative. Solving for ρ_{Rf} and ρ_{Rz} from eqs. (A1.1) and (A1.2) yields:

$$\rho_{Rf} = \frac{\sigma_f \hat{h}_{f0}}{\sigma_R} \tag{A2.1}$$

$$\rho_{Rz} = \frac{K\sigma_z \hat{h}_{0z}}{\sigma_R} \tag{A2.2}$$

Inserting eq. (A2.1) into the risk minimizing dual hedge (2.1) yields:

$$\hat{h}_f = \frac{1}{(1 - \rho_{fz}^2)}\left(\hat{h}_{f0} - \frac{\sigma_R}{\sigma_f}\rho_{fz}\rho_{Rz}\right) \tag{A3}$$

which implies:

$$\hat{h}_f - \hat{h}_{f0} = \rho_{fz}^2\hat{h}_f - \frac{\sigma_R}{\sigma_f}\rho_{fz}\rho_{Rz} \tag{A4}$$

Substituting $\hat{h}_f$ on the right-hand side of eq. (A4) by eq. (2.1) and collecting terms gives the desired result in eq. (7.1). Expression (7.2) can be obtained in the similar fashion.

APPENDIX B

The variance of revenue under bivariate normality of p and y can be decomposed using the moment generating function (Steen, 1982, p. 162, 177):

$$\mathrm{Var}(pXy) = X^2\{E(py)^2 - [E(py)]^2\} \tag{B1}$$

$$M_{p,y}(r, s) = \exp\left[(r\mu_p + s\mu_y) + \frac{1}{2}(r^2\sigma_p^2 + 2rs\sigma_{py} + s^2\sigma_y^2)\right] \tag{B2}$$

$$E(p^2y^2) = \left.\frac{\partial^4 M(r, s)}{\partial r^2 \partial s^2}\right|_{(r=s=0)}$$

$$= \sigma_p^2\sigma_y^2 + \mu_y^2\sigma_p^2 + 2\sigma_{py}^2 + 4\mu_p\mu_y\sigma_{py} + \mu_p^2\sigma_y^2 + \mu_p^2\mu_y^2 \tag{B3}$$

$$[E(py)]^2 = \sigma_{py}^2 + 2\mu_p\mu_y\sigma_{py} + (\mu_p\mu_y)^2 \tag{B4}$$

$$\mathrm{Var}(pXy) = X^2(\sigma_p^2\sigma_y^2 + \mu_y^2\sigma_p^2 + \sigma_{py}^2 + \mu_p^2\sigma_y^2 + 2\mu_p\mu_y\sigma_{py}) \tag{B5}$$

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