Threshold Effects In Panel Data Stochastic Frontier Model of Dairy Production In Canada

Abstract

One of the most enduring problems in econometrics is how to properly account for heterogeneity among firms. Threshold regression models are intuitively appealing methods to deal with this issue. We consider a fixed-effects panel data stochastic frontier model (Schmidt and Sickles (1984), Martin-Marcos and Suarez-Galvez (2000)) and, relying on Hansen (1999, 2000*a*), we propose an estimator that accommodates multiple thresholds. Our model assumes absence of any unmeasured time invariant heterogeneity across firms as in Greene (2005, p. 277). Slope and threshold parameters can be estimated using a within estimator combined with a grid search over the threshold parameters. Testing for threshold effects is problematic because threshold parameters are not identified under the null hypothesis, a case of the so-called Davies' problem. We apply the bootstrap procedure proposed by Hansen (1999, 2000*a*) to test for the presence of thresholds. An asymptotic confidence set for the threshold parameter can be obtained by inverting an LR test, using the distribution result presented in Hansen (1999, 2000*a*). Our empirical application features a panel of Quebec dairy farms. We use farm size as the threshold variable. The presence of a trend in the specification matters for the determination of the number of thresholds. Technical efficiency scores and rankings of farms estimated from competing model specifications are highly correlated and do not vary significantly across groups of farm sizes defined by the threshold parameter values.

Key words: Stochastic frontier models; threshold regression; technical efficiency; bootstrap; dairy production.

Journal of Economic Literature classification: C12, C13, C23, C52.

1 Introduction

Structural change and threshold effects are two related issues that have motivated considerable empirical and theoretical research in time series econometrics (e.g. Tsay (1989, 1998), Enders and Granger (1998), Hansen (2000*b*, 2000*a*)). An important problem in cross-sectional or panel data models is heterogeneity among firms. One approach to address this issue is to compare a regression function that is identical across all observations in a sample to a set of regression functions that allow for observations to fall into discrete classes as in Hansen (1999). With a focus on the dairy sector, we implement a stochastic frontier technical efficiency analysis that controls for heterogeneity among firms using a multiple-regime threshold production function.

Threshold regression models are intuitively appealing econometric methods to account for heterogeneity. In the context of stochastic production frontier models, the question may be whether larger firms use a production technology that differs from that of smaller firms. This would allow researchers to determine whether the higher productivity of large firms stems from the use of a different technology or simply a more efficient use of inputs given the constraints imposed by a common technology (see Tran and Tsionas (2006)). Related methods that allow for heterogeneity in stochastic frontier models include latent class models (Greene (2002, 2005); Orea and Kumbhakar (2004)), random coefficients models (Tsionas (2002); Greene (2002, 2005)) and Markov switching frontier models (Tsionas and Kumbhakar (2004)). The distinguishing feature of threshold models is that they assume that heterogeneity is induced by an observable exogenous variable, as opposed to unobservable random terms.

Recently, Tsionas and Tran (2006) have proposed various models that allow for heterogeneity in technology and in the distribution of technical inefficiency. Bayesian inference methods are proposed for the estimation of these models and for model comparisons.

Bayesian tools such as the posterior odds ratio and the Bayes factor are proposed for model selection, including the comparison of a threshold model against a model without threshold effects. These statistics are used as evidence pertaining to the presence of threshold effects in the data. However, from a classical inference approach, such evidence needs to be based on a statistical test. Testing for threshold effects is problematic and requires non standard tools because of the presence of a nuisance parameter which is not identified under the null hypothesis. This is known as Davies' problem and appropriate techniques have been proposed in Davies (1987), Andrews (1993) and Hansen (1996, 1999, 2000*a*). For our specific threshold effects problem, the nuisance parameter is the value of the threshold. In this paper, we consider a simple threshold stochastic frontier model. Relying on Hansen (1999, 2000*a*), we propose an estimation method using a grid search over the threshold parameter, and we provide a method to test the presence of one or more thresholds in a parametric stochastic frontier model with panel data. We consider a panel data stochastic frontier model with fixed effects time invariant inefficiency term. That is, we assume that the technical inefficiency term is a firm-specific constant, as in the fixed effects and random effects panel data models of Schmidt and Sickles (1984), Horrace and Schmidt (1996), Greene (1997) and Martin-Marcos and Suarez-Galvez (2000). However, because the statistical inference methods we use were specifically proposed for fixed effects panel data models, we do not consider the stochastic frontier models with random inefficiency terms. This formulation of the panel data stochastic frontier model has the advantage that it does not require any distributional assumption for technical inefficiency.

The paper illustrates that methods discussed in Hansen (1999, 2000*a*) have wide-ranging empirical applications in the analysis of technical efficiency using panel data. We report results from an empirical application that involves a panel of 302 dairy farms located in the province of Quebec and observed during 11 years, over the period 1993-2003. For this application, the threshold variable is the number of cows, a proxy for farm size.

The rest of the paper is organized as follows. Section 2 first describes how the milking technology used in dairy production in Quebec may give rise to heterogeneity among dairy farms, and then sets the statistical framework for the threshold stochastic frontier model. The Estimation method is presented in Section 3 while section 4 describes the method to test for the presence of thresholds. Section 5 focuses on the methodology proposed to construct a confidence set for the threshold parameters. Section 6 presents the results from our empirical application. The concluding section summarizes our results and their policy implications.

2 Thresholds in dairy production and statistical framework

This section first describes how threshold effects may arise in the dairy production technology and then presents our statistical framework for estimating technical efficiency scores.

2.1 Input lumpiness, input fixity and thresholds in dairy production

Lumpiness and fixity are common traits of inputs used in agriculture and dairy production in Canada is no exception. For example, dairy production in Canada requires heated dairy barns and the whole barn must be heated whether the barn is completely filled or half filled. Once the barn is filled and the maximum number of cows is reached, dairy production can only be increased through higher output per cow which can be achieved by increasing other inputs.

Figure 1: Input fixity and lumps

In Figure 1, the only way to change the level of output Y (or to move from one isoquant to another), requires changing the level of input X_2 since input X_1 is fixed at $\bar{x}_1$. In such a case, the expansion path is a straight vertical line. The production function can be written as $y = f(\bar{x}_1, x_{2,...}, x_n)$ and the cost function as $\omega_1 \bar{x}_1 + C(\omega_2, ..., \omega_n, y, \bar{x}_1)$ where ω_i is the price of input i. Naturally, barns come in different sizes, but to the extent that there are "common" sizes and that small changes in capacity are relatively too costly to be worth doing, we can expect that most dairy farms operate close to their capacity limits and that increases in size from a constrained equilibrium come in large lumps. Clearly, lumps and capacity constraints can also arise because of milking systems.

Other inputs, like tractors, can provide various levels of services up to a maximum capacity. The cost of such inputs is increasing up to their maximum capacity. Figure 2 illustrates two expansion paths associated with two tractors. The smaller tractor has a lower capacity, but it is more fuel efficient than the bigger one. Hence, the slope of its expansion path emanating from the origin has a flatter slope. The small dotted lines represent isocost lines. The steeper ones reflect the higher fuel costs of the larger tractor. Figure 2 clearly shows that it costs more to produce a low level of output Y_0 with a larger tractor than with a smaller one. By definition, an expansion path is defined by the equality of the marginal rate of technical substitution and the input price ratio times minus one. For the farm with the small tractor, this occurs at the tangency of the isoquant and the lowest of the flatter isocosts. For the farm with the larger tractor, this occurs at the tangency between the isoquant and the lowest steeper isocost lines. A flatter isocost line passes through this point of tangency and given that it lies everywhere above the isocost line associated to the production of Y_0 with the small tractor, we conclude that it is more expensive to produce small quantities of output with too large a tractor.

Figure 2: Lumpy and capacity constrained inputs

However, it is obvious that when the small tractor has reached its capacity limit, the large tractor may become a cheaper alternative. This is what happens at a level of output like

Y_1. The farm with the large tractor is not capacity-constrained as shown by the tangency between the isocost line and the isoquant. The farm with the small tractor is severely constrained at that level of production and must compensate by using more of the other inputs.

Different milking systems are used in dairy production. Because of their small size, many dairy herds in Quebec and Ontario, the two largest dairy producing provinces in Canada, are housed in tie stalls. This technology is labor intensive, but it does not require a large capital investment. Larger farms tend to use different variations of free stalling. In most cases, milking is done at fixed hours twice every day. However, new voluntary milking systems are being used by larger dairy farmers. These systems were first introduced in Europe, where grazing space is more limiting. Basically, the cows stay inside barns and get milked with robotic technology at the time they choose. The typical voluntary milking unit in Europe is for 60-70 cows (Wikipedia (2008)). In Ontario, the average user has 94 cows, even though such a system reportedly allows the family farm to expand up to 150 cows without hiring outside labor (Ontario Ministry of Agriculture, Food and Rural Affairs (2008)). Clearly, there are lumps and capacity constraints associated with milking systems.

Typical estimation of technical efficiency scores relies on a homogenous production function to define the efficiency frontier. The above discussion clearly shows that the lumpiness, capacity constraints and fixity of several key inputs introduce heterogeneity in milk production and this heterogeneity is intuitively driven by the size of the dairy herd.

2.2 Statistical framework

We consider the following two-regimes threshold stochastic frontier production model for the dairy sector:

$$y_{it} = \alpha + \beta_1' x_{it} I\left(q_{it} \leq \gamma\right) + \beta_2' x_{it} I\left(q_{it} > \gamma\right) - u_i + v_{it}, \quad u_i \geq 0, \tag{1}$$

where for firm i at time period t, $i = 1, ..., N$, $t = 1, ..., T$, y_{it} is the logarithm of output, $x_{it} \in \mathbb{R}^k$ is a vector of logarithm of inputs, $I(.)$ is the indicator function, β_1 and β_2 are two vectors of parameters associated with two different technologies Γ_1 and Γ_2; v_{it} is statistical error term, and $u_i \geq 0$ represents technical inefficiency. We assume throughout that the error term v_{it} is independent and identically distributed with mean zero and finite variance σ_v^2. q_{it} is an exogenous and observable threshold variable that governs the technology regime of firms in that sector; γ is the threshold value such that at time t, firms for which $q_{it} \leq \gamma$ adopt technology Γ_1 whereas all the other ones adopt technology Γ_2. To motivate this formulation with threshold effects, consider some stylized facts about dairy production. Even though there is a high proportion of small dairy farms, not all of the farms use the same milking system. Some farms are large enough to mix their feed on the farm. Some have little land or are located in areas where it is difficult to produce corn. Hence, it is justified to entertain the possibility that farms need not have the exact same technology. We may therefore posit that technological jumps occur at various farm sizes and accordingly production frontier models with one or more thresholds may be considered. For $\beta_1 = \beta_2$, we get the basic panel data stochastic frontier model (see Pitt and Lee (1981), Schmidt and Sickles (1984), Cornwell and Schmidt (1995), Greene (1997), and Martin-Marcos and Suarez-Galvez (2000)). An assumption that underlies our threshold effects formulation is that firms elect to switch from the Γ_1-technology to the Γ_2-technology because the cost of the new technology is lower than the loss in profit that firms would incur if they were not to switch. The time invariance assumption for the technical inefficiency term u_i may be an unreasonable one in long panels; we refer to Kumbhakar (1990) where it is argued that firms aware of their relative inefficiency would take steps to catch-up over time. This is less of a concern in our sample because all of the farms belong to management clubs which makes for more stable individual efficiency levels and rankings. Furthermore, the time invariant technical efficiency frontier model has a long history in the panel data stochastic frontier literature and it can be easily integrated in a multiple threshold framework. As Kumbhakar and Lovell (2000, p. 99) put it, the fixed-effects model has the virtue of simplicity and nice consistency properties. For further reference, see Pitt and Lee (1981), Schmidt and Sickles (1984), Greene (1997), Horrace and Schmidt (1996), Martin-Marcos and Suarez-Galvez (2000), Kim, Kim and Schmidt (2006) and references therein.

As in Hansen (1999), this model can be written in a more compact form as

$$y_{it} = \alpha + \beta' x_{it}(\gamma) - u_i + v_{it}, \quad u_i \geq 0, \tag{2}$$

where $x_{it}(\gamma) = \begin{pmatrix} x_{it} I(q_{it} \leq \gamma) \\ x_{it} I(q_{it} > \gamma) \end{pmatrix}$, $\beta = (\beta_1', \beta_2')'$. In this paper, estimation and inference in this model rely on Hansen (1999, 2000*a*). We assume that u_i is a fixed time invariant effect, $u_i \equiv \mu_i$, for all $t = 1, ..., T$.[1] This model assumes the absence of any unmeasured time invariant heterogeneity across firms (for further details we refer to Greene (2005, p. 277)). We write this model as a fixed effects panel data model as follows. Let $\alpha_i = \alpha - \mu_i$; then $\alpha_i \leq \alpha$ for all i and α_i may take positive or negative values, and we get the following non-dynamic panel model with firm-specific fixed effects:

$$y_{it} = \alpha_i + \beta' x_{it}(\gamma) + v_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T. \tag{3}$$

3 Estimation

The stochastic frontier model in (3) is the standard threshold regression for non-dynamic panel with individual-specific fixed effects discussed by Hansen (1999). Estimates for threshold and slopes parameters can be obtained using a "within" or least squares dummy variable estimator combined with a grid search for the threshold parameter. Specifically, estimation proceeds as follows. Assume that γ is known and let

$$\overline{y}_i = T^{-1}\sum\nolimits_{t=1}^{T} y_{it},\ \overline{x}_i(\gamma) = T^{-1}\sum\nolimits_{t=1}^{T} x_{it}(\gamma),\ \overline{v}_i = T^{-1}\sum\nolimits_{t=1}^{T} v_{it}, \quad i = 1, ..., N.$$

Apply a fixed effects transformation to (3) in order to remove firm-specific means and get

$$y_{it}^* = \beta' x_{it}^*(\gamma) + v_{it}^*, \tag{4}$$

where

$$y_{it}^* = y_{it} - \overline{y}_i,\ x_{it}^*(\gamma) = x_{it}(\gamma) - \overline{x}_i(\gamma),\ v_{it}^* = v_{it} - \overline{v}_i, \quad i = 1, ..., N, \quad t = 1, ..., T.$$

Model (4) can be written in matrix form as

$$Y^* = X^*(\gamma)\beta + v^*, \tag{5}$$

where $Y^*,\ X^*(\gamma)$ and v^* are the data stacked over all N firms and over T time periods as follows: for Y^*, form $Y^* = (y_1^*, ..., y_N^*)'$ where $y_i^* = (y_{i1}^*, y_{i2}^*, ..., y_{iT}^*)'$; proceed similarly to obtain $X^*(\gamma)$ and v^*. From (5), the ordinary least squares estimator of β (as a function of γ) is given by

$$\hat{\beta}_F(\gamma) = \left[X^*(\gamma)' X^*(\gamma)\right]^{-1} X^*(\gamma)' Y^*,$$

and the residual sum of squares is

$$\begin{aligned} S_F(\gamma) &= \left[Y^* - X^*(\gamma)\hat{\beta}(\gamma)\right]' \left[Y^* - X^*(\gamma)\hat{\beta}(\gamma)\right] \\ &= Y^{*\prime}\left(I - X^*(\gamma)' \left[X^*(\gamma)' X^*(\gamma)\right]^{-1} X^*(\gamma)'\right) Y^*. \end{aligned} \tag{6}$$

Since γ is unknown, it must be estimated from the data set. Estimate of γ can be defined as the value of γ with the minimum residuals sum of squares,

$$\hat{\gamma}_F = \arg \min_{\gamma \in \bar{\Gamma} \subset \Gamma} S_F(\gamma) . \tag{7}$$

The minimization in (7) can be restricted to a specific subset $\bar{\Gamma} \subset \Gamma$, where Γ is the set of all possible values of γ, if we want a minimal percentage of the observations to lie in each of the two technology regimes defined by the threshold. A grid search over values in $\bar{\Gamma}$ is used in practice to solve this problem; see Hansen (1999, pp. 349-350) for details. The final estimate of the regression coefficients β is $\hat{\beta}_F = \hat{\beta}_F(\hat{\gamma}_F)$; the vector of residuals is $\hat{v}^*_F = Y^* - X^*(\hat{\gamma}_F)\hat{\beta}_F(\hat{\gamma}_F)$ and the error variance is estimated by $\hat{\sigma}^2_{vF} = (1/NT) S_F(\hat{\gamma}_F)$. Technical efficiency scores are estimated following the strategy proposed in Schmidt and Sickles (1984): $\hat{\alpha}_i = (1/T)\sum_{t=1}^{T} \hat{v}^*_{F,it}$, $\hat{\alpha} = \max_i(\hat{\alpha}_i)$, $\hat{u}_i = \hat{\alpha} - \hat{\alpha}_i$, $TE_i = \exp(-\hat{u}_i)$, $i = 1, 2, ..., N$ (see also Horrace and Schmidt (1996) and Martin-Marcos and Suarez-Galvez (2000)).

4 Testing for threshold effects

Model (1) and the estimation methods discussed in the previous sections assumed that there exists some threshold effect in the data. However, since this formulation introduces an extra (threshold) parameter in the model, estimation problems may arise due to a specification error when there is actually no threshold effects in the data. Therefore, it is important to assess the plausibility of one or more thresholds using a formal statistical test. To this end, we also rely on the likelihood ratio test proposed in Hansen (1999), which we outline here for convenience. The null hypothesis of no threshold effect in the model (1) is $H_0 : \beta_1 = \beta_2$.

Clearly, under H_0 the model (1) takes the form

$$y_{it} = \alpha + \beta_1' x_{it} - u_i + v_{it}, \quad u_i \geq 0, \tag{8}$$
$$i = 1, ..., N, \quad t = 1, ..., T,$$

which does not involve the threshold parameter γ. So for the problem at hand, the parameter γ is not identified under the null hypothesis. This is the so-called Davies' Problem (Davies (1977, 1987)) and usual test statistics have non-standard distributions. For this problem, Hansen (1999) suggested to simulate the non-standard asymptotic distribution of the likelihood ratio (LR) test statistic using a bootstrap method as follows.

We estimate the fixed effects panel data stochastic frontier model associated to model (8) using the "within" estimator. Let y_{it}^*, x_{it}^*, and v_{it}^* denote the within transformation versions of y_{it}, x_{it}, and v_{it} respectively. For reference, let $\tilde{\beta}_{1F}$ denote the within estimator of β_1. Let $\tilde{v}_F^*$ denote the vector of residuals and $S_{0F} = (\tilde{v}_F^*)' (\tilde{v}_F^*)$ be the residuals sum of squares under H_0.The LR test statistic (see Hansen (1999)) is defined as

$$LR_F = (S_{0F} - S_F(\hat{\gamma}_F)) / \hat{\sigma}_{vF}^2. \tag{9}$$

We rely on the bootstrap procedure proposed by Hansen (1999) for the standard fixed effects panel model. The resampling is based on the sample of firms, so that once a firm is selected all its observations over the T periods are included in the bootstrap sample. We resample residuals as follows. Let $\hat{v}_{F,i}^* = \left(\hat{v}_{F,i1}^*, \hat{v}_{F,i2}^*, ..., \hat{v}_{F,iT}^*\right)'$, $i = 1, ..., N$, denote the $T \times 1$ vector of residuals computed for firm i from the model assuming threshold effects. Then form the sample $\left(\hat{v}_{F,1}^*, \hat{v}_{F,2}^*, ..., \hat{v}_{F,N}^*\right)$. The empirical distribution of $\left(\hat{v}_{F,1}^*, \hat{v}_{F,2}^*, ..., \hat{v}_{F,N}^*\right)$ is used for bootstrap resampling, *i.e.* we draw randomly with replacement a sample of size N from $\left(\hat{v}_{F,1}^*, \hat{v}_{F,2}^*, ..., \hat{v}_{F,N}^*\right)$. These draws are treated as errors to be used to create a bootstrap sample under H_0. For each bootstrap replication $b = 1, ..., B$, let $\left(v_1^{(b)}, ..., v_i^{(b)}, ..., v_N^{(b)}\right)$ represents the bootstrap draw. We generate the output variable using

$$y_{it}^{(b)} = \hat{y}_{it} + v_{it}^{(b)},$$

where $\hat{y}_{it}$ is the predicted value of y_{it} under H_0: $\hat{y}_{it} \equiv \tilde{y}_{it}^* = \tilde{\beta}_{1F}' x_{it}^*$. Using the bootstrap sample data $\left(y_{it}^{(b)}, x_{it}\right)$, we estimate in turn the model under H_0 and without imposing H_0. We compute the bootstrap value $LR_F^{(b)}$ of the LR test statistic using (9). Let LR_F^0 denote the value of the test statistic calculated from the observed data; then we define the approximate bootstrap p-value $\widehat{p}_B\left(LR_F^0\right)$ as

$$\hat{p}_B\left(LR_F^0\right) = \frac{B\hat{G}_B\left(LR_F^0\right) + 1}{B + 1}, \tag{10}$$

where $B\hat{G}_B\left(LR_F^0\right)$ is the number of bootstrap statistics $LR_F^{(b)}$ greater than or equal to LR_F^0. A test of level α, $0 < \alpha < 1$, is defined by the critical region $\hat{p}_B\left(LR_F^0\right) \leq \alpha$; that is, we reject the null hypothesis at level α if $\hat{p}_B\left(LR_F^0\right) \leq \alpha$, $0 < \alpha < 1$.

5 Confidence set for the threshold parameter

In the presence of a threshold effect, it would be useful to provide a confidence set for threshold parameters as this would strengthen the interpretation of the results. Indeed, in the related time series structural change literature a confidence set for the break date is now commonly constructed and reported based on the asymptotic distribution of the estimator of the break date (see Bai, Lumsdaine and Stock (1998)).

The threshold parameter γ is not identified asymptotically and the asymptotic distribution of its estimator $\hat{\gamma}$ is highly non-standard (Hansen (2000a)). In such contexts, Wald or

t statistics-based confidence sets may not be reliable, particularly in finite sample (Dufour (1997), Seo and Linton (2007)). Hansen (2000a) recommended confidence set estimation based on inverting likelihood ratio tests on γ. Inverting a test with respect to a parameter is equivalent to assembling all the values of this parameter for which the test is not significant. So we consider the test of the hypothesis $H_0(\gamma_0) : \gamma = \gamma_0$, where γ_0 is any specified value for γ. The LR statistic to test $H_0(\gamma_0)$ is

$$LR_F(\gamma_0) = (S_F(\gamma_0) - S_F(\hat{\gamma}_F)) / \hat{\sigma}^2_{vF}. \tag{11}$$

Hansen (1999, 2000a) shows that in the case of the fixed effects model the asymptotic distribution of $LR_F(\gamma_0)$ under $H_0(\gamma_0)$ is non-standard and free of nuisance parameters. Under regularity conditions, $LR_F(\gamma_0) \overset{asy}{\sim} \omega$, where ω is a random variable with distribution function $P(\omega \leq x) = (1 - \exp(-x/2))^2$. The critical value of the latter distribution at level α, $0 < \alpha < 1$, is $c(\alpha) = -2\ln\left(1 - \sqrt{1-\alpha}\right)$. An asymptotic test of $H_0(\gamma_0)$ rejects at level α if $LR_F(\gamma_0) > c(\alpha)$. A $(1-\alpha)$-level confidence set for γ is therefore defined by the "no-rejection region" of the LR test as

$$CS(\gamma;\alpha) = \{\gamma_0 : LR_F(\gamma_0) \leq c(\alpha)\}. \tag{12}$$

The asymptotic validity of this confidence set requires, among other conditions (Hansen (2000a, p. 579)), that the difference in the slope parameters between the two regimes be small and tend to zero as the sample size increases. This confidence set is rather asymptotically conservative if the error terms v_{it} are $i.i.d.$ $N(0, \sigma^2_v)$ and independent of the regressors and of the threshold variable (see Hansen (2000a, Theorem 3)).

The methodology presented so far focused on a single threshold parameter allowing for two regimes or production technologies. However, it is easy to accommodate multiple thresholds and conveniently adapt previous inference procedures. Indeed, using results from the multiple changepoint model, Hansen (1999, section 5) suggests a refinement estimator for the double-threshold model. This refinement estimator requires sequential estimation and is shown to be consistent and asymptotically efficient in the changepoint model. The adaptation of the bootstrap LR test for zero threshold against one to the case of $m-1$ thresholds versus m, $m > 1$, using the refinement estimation procedure is straighforward. Hansen (1999, section 5) also discusses how to obtain confidence intervals for $m > 1$ threshold parameters through inversion of LR statistics based on refined sum of squared errors. For further details on these extensions we refer to Hansen (1999, section 5).

6 Empirical application

In this section we report results from an application of the methods discussed above to an empirical data set featuring a panel of dairy farms located in the province of Quebec.

6.1 Data sources and descriptive statistics

We consider a balanced panel covering 11 annual observations for 302 dairy farms that were in business between 1993 and 2003. Thus, our data set has a total of 3322 observations. This so-called Agritel database was collected by the Federation of Management Clubs in the province of Quebec. Summary statistics on the different variables used in our stochastic frontier production models and the threshold variable are presented in Table 1.

Canada's dairy production is governed by a supply management policy featuring tight import controls and domestic production quotas to insure a "fair" return for dairy producers. Basically, supply is constrained to achieve a domestic price target (Larue, Gervais and Pouliot (2007)). Individual production licences or quotas are traded between producers within the province of Quebec through a double-auction. The value of these individual quotas has steadily increased over time and represents a significant financial barrier deterring entry and expansion. This explains why the average number of cows is low compared to U.S. standards and why there are so few large dairy farms in Quebec[2]. The inputs selected as arguments of the production function are the most important ones in terms of cost shares. The standard deviations are much smaller than the means because there is a significant proportion of farms that are quite similar size-wise. We begin our investigation with a fixed effects stochastic frontier model without threshold effects.

6.2 A stochastic production frontier with a homogenous technology

The fixed effects stochastic frontier model without threshold can be considered as our benchmark. We estimated four different versions to assess the robustness of the results. We consider two different functional forms for the production technology which could be specified with or without a trend. The most popular functional forms used in the applied literature are the Cobb-Douglas and the Translog. The latter is more flexible than the former, but it involves the estimation of more parameters. The presence of a trend allows for technological change. The summary statistics for estimated technical efficiency scores derived from the four competing specifications are presented in Table 2. Our results suggest that the choice of the functional form does not have much influence on the central tendency and dispersion statistics of the (time-invariant) efficiency scores. The mean and median are very close to 96% in all cases. The standard deviations are very small, which is not surprising given that the minima vary between 94% and 95%. Such high efficiency scores for Quebec dairy farms are to be expected because the supply management policy has been in place for a long time and, despite all of its flaws, it cannot be denied that it has contributed to create a stable environment for dairy farmers. Technical efficiency is a relative concept since the frontier is defined using the firms included in the sample. The Quebec dairy industry is subject to far less volatility than the U.S. dairy industry and this should make management easier. Finally, our data set is made up of farms belonging to management clubs and they probably get similar dairy herd management advice.

6.3 A stochastic production frontier with threshold effects

Even though Quebec has a high proportion of small dairy farms, not all of the farms use the same milking system. Some farms are large enough to mix their feed on the farm. Some have little land or are located in areas where it is difficult to produce corn. Hence, it is justified to entertain the possibility that farms need not have the exact same technology. In this section, we posit that technological jumps occur at various farm sizes and we consider production frontier models with one or more thresholds. The threshold variable is the farm size as measured by the number of cows on the farm. We find numerically the least squares estimates of the threshold parameters through a grid search over 500 quantiles of the empirical distribution of the threshold variable[3]; we trimmed out top and bottom 1% in order to eliminate outlying farm size values.

In our application, we allowed for up to three thresholds supporting four different regimes. Table 3 reports test results pertaining to the number of thresholds. Under the null hypothesis, the model has $m-1$ thresholds while the alternative has m thresholds. The presence of a trend in the specification makes a huge difference and in both the Cobb-Douglas and Translog cases, the empirical evidence supports three thresholds. For the Translog without trend, there is apparently only one threshold (interpreting a p-value of 0.08 as a rejection at the 10% level). For the Cobb-Douglas case without trend, the tests results suggest the absence of any threshold value in the model.

The point estimates for the threshold parameters are presented in Table 4 along with lower and upper bounds of the corresponding 95% confidence sets for the Cobb-Douglas and Translog forms with and without a trend. There is no threshold point estimate for the Cobb-Douglas model without trend. The Cobb-Douglas frontier with trend has three thresholds whose point estimates are 34, 45 and 66. All three thresholds have narrow confidence sets that do not overlap. The point estimates obtained from the Translog with a trend are nearly identical to the ones reported for the Cobb-Douglas case, but the confidence sets differ. In this instance, the confidence set for the first threshold is very narrow while the second and third thresholds have low lower bounds that result in overlapping. The Translog frontier without a trend supports a single threshold. The latter's point estimate is 48 with a lower bound of 46 and an upper bound of 49. Some of these confidence sets are asymmetric around the point estimate, which is not a problem with confidence sets constructed by inverting a test. This is also apparent in Hansen (1999), but to a lesser degree. Even though the number of thresholds varies between the models, there is overlap in the set estimates of threshold parameters.

Table 5 reports estimates of the coefficients characterizing the production technologies of the four regimes associated with the Cobb-Douglas with trend frontier. The concentrate coefficients vary between 0.095 and 0.161 across regimes while the range for the forage coefficients is 0.031-0.059. The coefficients on capital are small and not significantly different from zeros for the three smallest categories of farms. In contrast, labour is most important for the smallest farm group. The labour coefficient for the smallest farms is roughly 50% larger than that for the largest farms. The trend coefficients are very similar across regimes.

Results relative to efficiency scores estimated using the threshold effects models are presented in Table 6. The mean efficiency level is close to 96% in all cases, very similar to the results obtained for the stochastic frontier models without threshold effects. This result suggests that productivity advantage of larger dairy farms over smaller farms are due to technological differences and not to differences in technical efficiency. Table 7 reports the most and least technically efficient farms for the model formulations considered. From this table, it can be seen that the same firm is identified as the most technically efficient no matter the model formulation we consider; this holds for models with or without threshold effects. Things are less clear about the least technically efficient firms because our models identify three different firms. All models with a trend are unanimous, while Cobb-Douglas models without trend, with or without threshold effects, are in agreement. Correlations or rank correlations coefficients between technical efficiency scores estimated from the considered model formulations are reported in Tables 8. These results show relatively high correlation coefficients between estimated efficiency scores, regardless of the functional form chosen to depict the technology, the presence or absence of a trend and the number of identified thresholds. Taken together, our results are robust to changes in model specifications which is in line with Kumbhakar and Lovell's (2000, p. 90) observation about the robustness of cross-sectional technical efficiency scores to distributional assumptions.

The next issue of interest in our application is the empirical distribution of estimated efficiency scores within regimes defined by the threshold parameters. In Table 9 we report p-values for analysis of variance (ANOVA) tests on estimated efficiency scores.[4] The null hypothesis of these tests is that the means of efficiency scores are equal across regimes. As may be seen from these Tables, for most years covered by our data we can not reject the null hypothesis at the 5% significance level; we get rejections for all threshold effects models only in the last year of our data. We can conclude that technical efficiency scores do not vary significantly, at the 5% level, across regimes or specific groups of farm sizes as defined by the threshold parameters values. Nethertheless, it appears that for the last year of our data set, groups of larger farms have higher mean technical efficiency scores than groups of smaller farms.

7 Conclusion

Heterogeneity among firms in cross-section or panel data models is an issue that has motivated a rapidly increasing literature. This problem may be present in data sets used for estimating technical efficiency in panel data stochastic frontier model framework. In this paper, we address the heterogeneity issue in the fixed effects stochastic frontier model. We rely on statistical inference methods developed in Hansen (1999, 2000*a*). These methods allow for the presence of multiple thresholds. Testing for the presence of threshold effects is problematic because of nuisance parameters not identified under the null hypothesis. We also use the test inversion method proposed in Hansen (1999, 2000*a*) to obtain confidence intervals for the threshold parameters.

Our empirical application features the estimation of fixed effects stochastic frontier model on a panel of 302 dairy farms located in the province of Quebec and observed during 11 years, over the period 1993-2003. We found evidence of threshold effects, but the number of thresholds is sensitive to the presence or absence of a trend and to the functional form used to represent production technology. The efficiency scores are highly concentrated at the top for models with or without thresholds, no matter the functional form used for the production technology, the inclusion of trend in the production function, or that the model assumes threshold effects or not. We also find that technical efficiency scores do not vary significantly, at the 5% level, across groups of farm sizes as defined by the threshold parameters values. From the 11 years covered by our data set, we find evidence of differences in mean levels of technical efficiency across regimes for the last year only, in which case higher technical efficiency should be associated with larger farm sizes. These results are conditioned by the unique characteristics of our sample. All of our 302 dairy farms belong to management clubs and their individual size has not changed very much over the 11 years of our panel. This lack of variation in size over time for any given farm and the sample range of farm sizes are policy-constrained. The supply management policy has made it difficult for dairy producers to expand. As a result, high and stable technical efficiency averages are to be expected, even though farms of different sizes are using different technologies.

The number of dairy farms has been declining and the average size increasing for a long time in spite of the high degree of protection. Trade liberalization would most likely exacerbate these trends. Our results suggest that factors like age and the capacity to borrow to increase milking capacity to US-like levels would probably be better predictors of exit decisions than technical efficiency scores.

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Notes

[1]The case of random time-invariant inefficiency terms is also discussed in the literature (Battese and Coelli (1988, 1992), Greene (1997)), but the statistical inference methods we use to account for threshold effects are established for fixed effects models and we are not certain if they remain valid for a random specific effects model. Moreover, a more flexible formulation that we do not discuss in this paper allows for inefficiencies to vary over time. This is an obvious advantage when dealing with long panels. For various formulations and specifications for the time dependence of technical inefficiency, see Cornwell, Schmidt and Sickles (1990), Kumbhakar (1990), Lee and Schmidt (1993) and Battese and Coelli (1992, 1995) among others; it is not clear if the methods discussed in Hansen (1999, 2000*a*) apply to these models.

[2]According to http://www.dairyfarmingtoday.org/DairyFarmingToday/Learn-More/Facts-And-Figures/ consulted on March 10, 2009, the average herd size in the U.S. is 135 cows. See also Romain and Sumner (2001) on comparisons between the Canadian and U.S. dairy industries.

[3]Our computations rely on appropriate adaptations of a Gauss code which is available on Bruce Hansen's homepage at http://www.ssc.wisc.edu/ bhansen.

[4]Tests based on their rankings gives very similar results.

Table 1. Summary Statistics For Dairy Production Variables

Variables	mean	Std. dev.	Min.	Max.
Production function:				
Volume of milk/cow (litre)	8304.03	1281.12	4557.87	12253.09
Concentrates (kg)	2879.73	741.77	632.30	6417.81
Forages (kg)	5273.25	949.78	390.44	9270.93
Capital ($)	4801.67	2545.28	372.84	34917.92
Labor (hour)	57.28	13.92	23.49	120.93
Threshold:				
Number of cows	51.64	25.58	18.70	451.90

Table 2. Summary Statistics For Estimated Technical Efficiency Scores Derived From A Production Frontier Without Threshold Effects

	Cobb-Douglas		Translog	
Statistics	No trend	Trend	No trend	Trend
Mean	96.03	96.64	95.69	96.58
Stand. dev.	.69	.65	.72	.64
Median	96	96.65	95.62	96.60
Minimum	94.27	95.09	94.04	95

Note. This table reports descriptive statistics for technical efficiency scores (in %) estimated in the framework of a panel data stochastic production frontier model with fixed effects inefficiency terms. The estimation method assumes that there is at least one fully efficient firm in the sample, so the maximum value is 100.

Table 3. Tests Of $m-1$ Thresholds Against m In A Stochastic Production Frontier Model: Bootstrap p-Values

	Cobb-Douglas		Translog	
m	No trend	Trend	No trend	Trend
1	.627	.004	.072	.004
2	.406	.001	.650	.004
3	.771	.006	.720	.008

Note. The numbers in this table are bootstrap p-values for the test of the null hypothesis that there exists $m-1$ threshold values for the production function against the alternative of m, $m = 1, 2, 3$. For an α level test, the null hypothesis is rejected if the reported p-value is less than or equal to α.

Table 4. Point Estimates And 95% Level Confidence Set For Threshold Parameters In A Threshold Stochastic Production Frontier Model

Parameter		Cobb-Douglas	Translog	
		Trend	No trend	Trend
	$\hat{\gamma}_1$	34.9	48.0	34.4
γ_1	γ_{1L}	34.1	46.1	34.1
	γ_{1U}	35.3	49.0	34.9
	$\hat{\gamma}_2$	45.1	-	44.7
γ_2	γ_{2L}	44.7	-	26.2
	γ_{2U}	50.0	-	45.5
	$\hat{\gamma}_3$	66.3	-	66.7
γ_3	γ_{3L}	65.6	-	44.7
	γ_{3U}	68.1	-	67.7

Note. This table reports the point estimates and the lower and upper bounds of 95% level confidence sets for the threshold parameters constructed by inverting an LR test statistic in a model with fixed effects inefficiency terms. The threshold parameters are $\gamma_1, \gamma_2, \gamma_3$; $\hat{\gamma}_i$, $i = 1, ..., 3$ denote the point estimate of γ_i; γ_{iL} and γ_{iU} respectively denote the lower and upper bounds of the 95% level confidence set for γ_i.

Table 5. Regression Coefficients Estimates: Triple Threshold Model For Cobb-Douglas Technology With A Trend

	regime 1		regime 2		regime 3		regime 4	
Variables	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
Concent.	.1185	7.57	.1495	14.22	0.1612	15.30	0.0950	5.14
Forages	.0487	3.75	.0317	3.87	0.0362	3.79	0.0588	4.72
Capital	-.0034	-.53	.0042	0.89	0.0029	0.67	0.0152	2.45
Labor	.0911	5.22	.0424	3.26	0.0150	1.11	0.0582	3.29
Trend	.0257	22.32	.0256	33.24	0.0205	25.89	0.0252	21.67

Note. This table reports coefficients estimates for a production frontier model with three thresholds values and fixed effects inefficiency terms; the production function relies on a Cobb-Douglas technology with a trend; t-ratios are based on White-corrected standard errors.

Table 6. Summary Statistics For Estimated Technical Efficiency Scores Derived From A Threshold Effects Stochastic Production Frontier Model

	Cobb-Douglas	Translog	
Statistics	Trend	No trend	Trend
Mean	96.68	95.93	96.64
Stand. dev.	.62	.73	.66
Median	96.72	95.84	96.67
Minimum	95.11	94.31	95.03

Note. This table reports descriptive statistics for technical efficiency scores (in %) estimated from a threshold panel data stochastic production frontier model. The number of threshold m can be read from Table 3. For the fixed effects model, the estimation method assumes that there is at least one fully efficient firm in the sample, so the maximum value is 100.

Table 7. Estimated Least And Most Efficient Farms (Best And Worst Farms) From Competing Model Specifications

	Cobb-Douglas		Translog	
	No trend	Trend	No trend	Trend
	Specifications without threshold effects			
Best farm	116	116	116	116
Worst farm	120	236	144	236
	Specifications with threshold effects			
m	0	3	1	3
Best farm	116	116	116	116
Worst farm	120	236	144	236

Table 8. Correlations Between Estimated Technical Efficiency Scores From Competing Model Specifications

a. Correlation Coefficients

	CDI0	CDIT0	TLI0	TLIT0	CDIT3	TLI1	TLIT3
CDI0							
CDIT0	.886						
TLI0	.964	.854					
TLIT0	.882	.996	.869				
CDIT3	.862	.967	.834	.965			
TLI1	.941	.848	.976	.863	.808		
TLIT3	.858	.958	.839	.961	.988	.820	

b. Rank Correlation Coefficients

	CDI0	CDIT0	TLI0	TLIT0	CDIT3	TLI1	TLIT3
CDI0							
CDIT0	.875						
TLI0	.966	.848					
TLIT0	.873	.996	.862				
CDIT3	.846	.957	.817	.953			
TLI1	.944	.834	.974	.848	.781		
TLIT3	.849	.949	.829	.949	.985	.794	

Note. This table reports the correlation matrix between estimates of technical efficiency scores obtained from different models and the correlation matrix between their ranks. The models labels are as follows: the first two letters pertain to the functional form describing the technology: "CD" for Cobb-Douglas and "TL" for Translog. The remaining letter characters indicate a trend is included (IT) or not included (I). The end number is the number of thresholds (0,1,2 or 3).

Table 9. Analysis Of Variance Tests For Differences In Technical Efficiency Scores Across Regimes

		Years										
Model	Groups	1	2	3	4	5	6	7	8	9	10	11
CDIT3	4	.217	.107	.055	.178	.155	.159	.518	.216	.284	.441	.012
TLI1	2	.295	.084	.310	.215	.192	.077	.081	.349	.056	.014	.002
TLIT3	4	.323	.286	.101	.316	.270	.103	.754	.318	.468	.549	.036

Note. This table reports p-values for the analysis of variance tests for the compairison of means of estimated technical efficiency. The groups used for the tests are technological regimes as implied by the estimated threshold parameters. The column entitled "Groups" indicates the number of different technological regimes or groups, which equals $m+1$, where m is the number of thresholds detected by our tests. The tests are conducted for each observation period given that the threshold variable is time variant and technical efficiency is assumed time-invariant. A p-value less than 5% implies that technical efficiency significantly differs across groups at the 5% level. We refer to Table 8 for labels used for the models.