A Second Year Course on an Introduction to Applied Mathematics

http://www4.ncsu.edu/eos/users/w/white/www/white/ma325.htm

by

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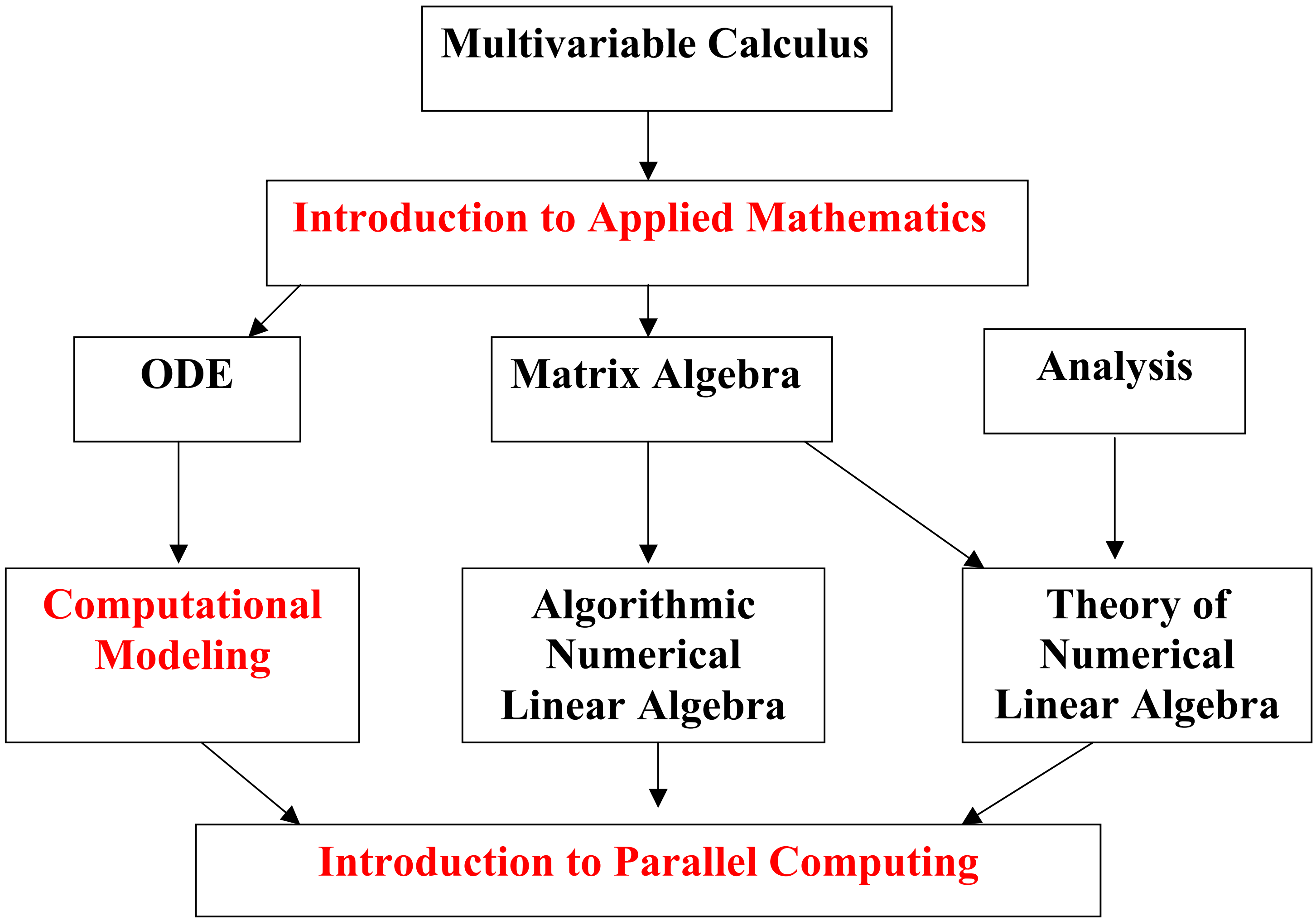

Multivariable Calculus
Introduction to Applied Mathematics
ODE
Matrix Algebra
Analysis
Computational Modeling
Algorithmic Numerical Linear Algebra
Theory of Numerical Linear Algebra
Introduction to Parallel Computing

Introduction to Applied Mathematics (MA 325)

- **A survey of applications of mathematics**
- **Suitable for students who have taken multivariable calculus**
- **Student to formulate a cohesive plan of study for the third and fourth years**
- **Five three-week modules on variety of applications**

Module Attributes

- **Mathematics as is needed**
- **Variety of applied mathematics**
- **Motivational**

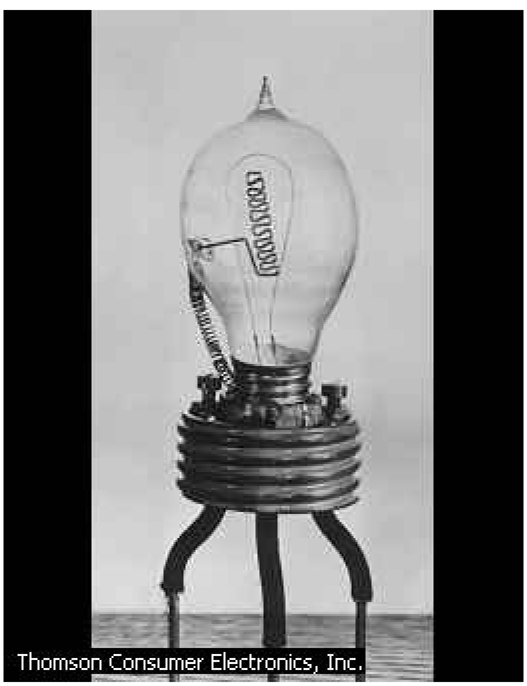
Module on Heat and Mass Transfer

- **Newton cooling and stability**
- **Discrete versus continuous models**
- **Analysis of discretization error**
- **Diffusion in a wire**
- **Analysis of stability**
- **Diffusion in a cooling fin**
- **Pollutant transfer in a stream**
- **Pollutant transfer in a lake**
- **Analysis of von Neumann series**

R. E. White

The ENIAC computer had over 18,000 vacuum tubes, and also it had a lot of heat…note the overhead fans.


UPI/THE BETTMANN ARCHIVE/Corbis


Thomson Consumer Electronics, Inc.

The Pentium CPU still needs to be cooled by heat sinks, cooling fins and fans.

The fabrication of metal objects such as sheet metal and metal beams required very high temperatures and heat loss via "radiation".


H.R. Bramaz/Peter Arnold, Inc.

Homes lose or gain heat via "diffusion" of heat.

Oil pumped from the earth is very hot. Much of the Alaskan pipeline in above frozen “mud”, and therefore, the hot oil may melt the foundation of the pipeline!

During reentry, a swiftly moving shuttle encounters drag from the atmosphere, which generates tremendous heat. To prevent the shuttle from being damaged, heat-resistant tiles cover the whole orbiter. The most vulnerable areas, such as the nose and leading edges of the wings, are covered with reinforced carbon-carbon tiles that can withstand temperatures of up to 1,430° C (2606° F).[1]

Can we predict the temperature of a mass as a function of time and space given some observed initial temperature, and boundary temperatures?

Math Model

- **From discrete Newton's cooling law**

$$u^{k+1} = u^k + \Delta t\, c(u_{sur} - u^k), \quad 1 - \Delta t\, c > 0$$

- **Gives geometric series**

- **Heat diffusion requires a generalization**

$$u^{k+1} = Au^k + d \text{ where } A \text{ is } n \times n$$

Ronald Toms/Oxford Scientific Films

Point source of a pollutant.

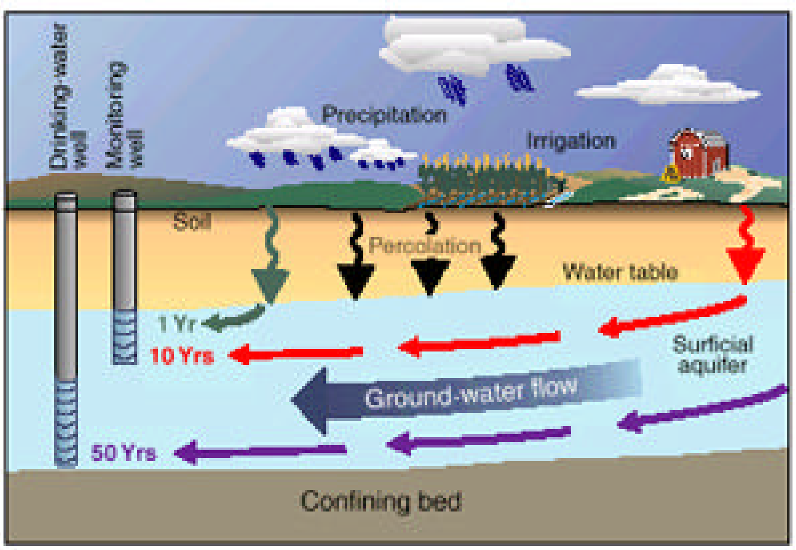
John Dommers/Photo Researchers, Inc.

Pollutants in lakes and streams.

Ben Osborne/Oxford Scientific Films

Pollutants in the air that are trapped by thermal inversion.

Pollutants in the soil "diffuse" into the water supply.



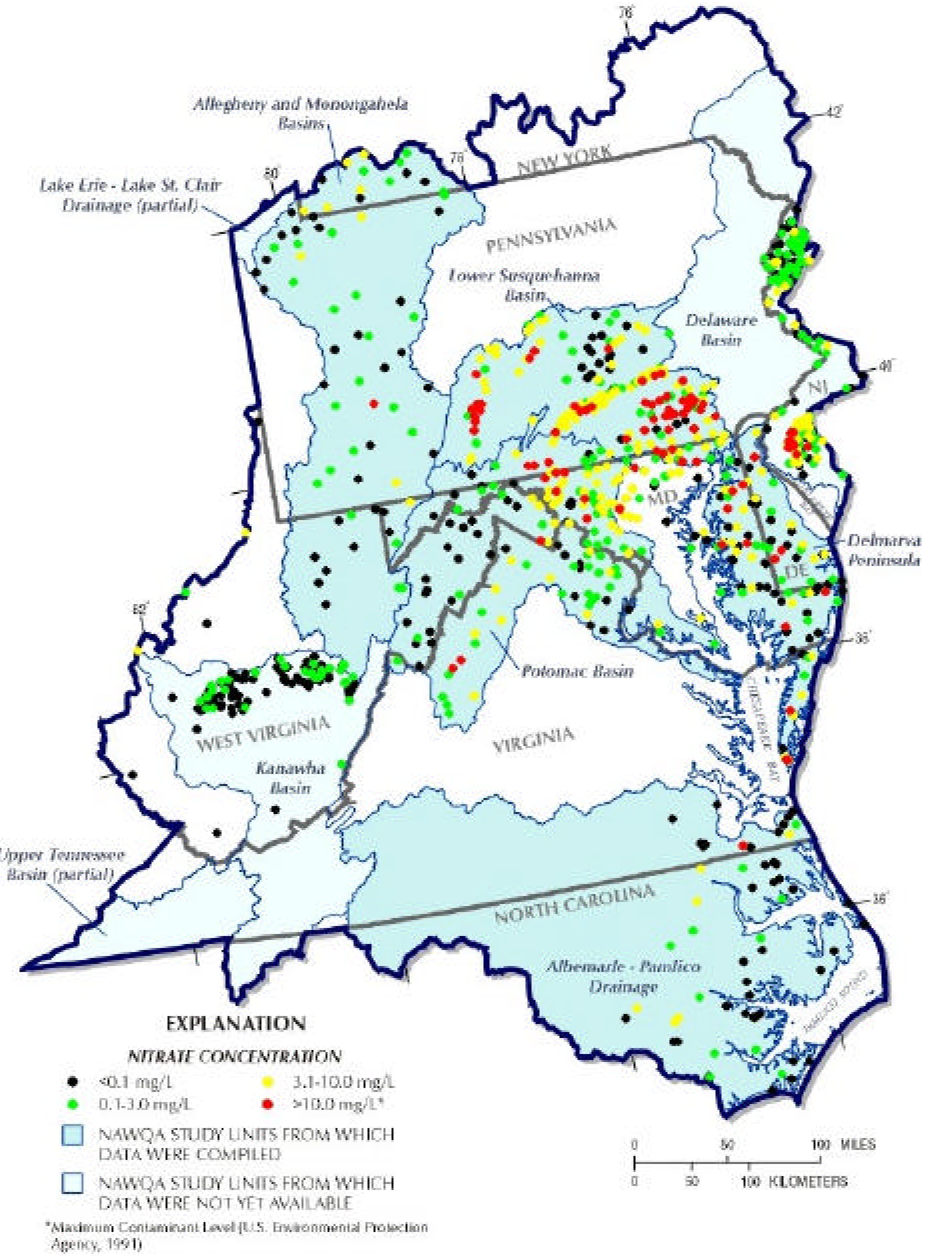


Figure 3. *Map of the Mid-Atlantic region showing location of available regional nitrate data and distribution of nitrate concentrations.*

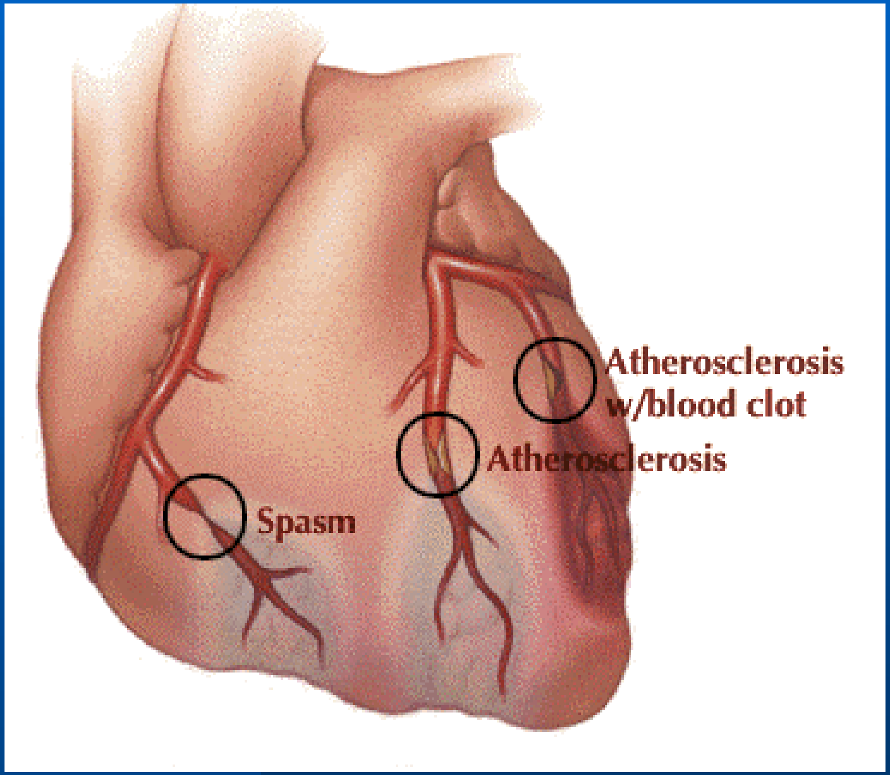
Can we predict the concentrations of pollutants as a function of space and time given some limited observations?

Module on Acoustic Waves and Boundary Conditions

- **Development of the wave equation from first principles**
- **Development continued**
- **Development of several types of boundary conditions**
- **Discuss experimental procedures and data collection**
- **Formulate the least square problem for parameter estimation**
- **Visit to the CRSC/Math laboratory for data collection**
- **Analysis of data**
- **Analysis continued**

H. T. Tran

Atherosclerosis



- Build-up of plaque (fat and calcium deposits) on the interior walls of the arterial lumen
- The most common disease of arteries

Turbulence Induced Acoustic Waves

- **Significant deposits will disrupt the blow flow**
  - Produce distinctive turbulence acoustic signatures

- **Large arteries (carotid arteries)**
  - Detected by a stethoscope
- **Small arteries deep inside the body, it is very difficult to detect**
  - Attenuation of acoustic energy through the intervening tissues
  - Other complex sounds with the body can overwhelm the acoustic detection systems

Opportunity for CAD Detection

- Unnecessary deaths
  - 500,000 deaths each year (55% w/o warning)
- Unnecessary diagnostic tests
  - 20-30% of angiogram patients are normal
  - Costing $2 billion each year
- Unnecessary hospital admissions
  - 600,000 chest pain patients/year admitted=normal
  - Costing $2 billion at $4,000 avg. cost per day

MedAcoustics: Acoustic Sensors

- Accurate
- Inexpensive
- Non-invasive
- Presents no risk to patient
- Easy to administer

Wave Equation

- **1-D wave equation for the velocity potential:**

$$\phi_{tt} = c^2 \phi_{xx}, \quad 0 < x < l$$

- **Boundary conditions?**
  - Damped harmonic oscillator

$$m\delta_{tt} + d\delta_t + k\delta = -\rho\phi_t(t,0)$$

  - Boundary surface is not penetrable by the fluid

$$\delta_t(t) = \phi_x(t,0)$$

HTB, GP, CW (1996)

Solution

$$\phi(t,x) = F(t - x/c) + G(t + x/c)$$

$$\delta_t(t) = \phi_x(t,0) \quad \Rightarrow \quad \delta(t) = -\frac{1}{c}(F(t) - G(t))$$

$$mG'' + (d + \rho c)G' + kG = mF'' + (d - \rho c)F' + kF$$

Assume incident wave is a simple harmonic $\Rightarrow$ $F(t - x/c) = e^{i\omega(t - x/c)}$

$$G(t + x/c) = R(\omega)e^{i\omega(t + x/c)} \quad \Rightarrow \quad R(\omega) = \frac{m\omega^2 - i(d - \rho c)\omega - k}{m\omega^2 - i(d + \rho c)\omega - k}$$

An Inverse Problem

- **Problem:**

  Given $R(\omega_i)$ **find the unknown parameters** (m, d, k, ρ) **in the damped harmonic oscillator boundary model**

- **Determine** $R(\omega)$?

  – The acoustic pressure for a planar wave propagation is given by

$$p(t,x) = A(\omega)e^{i\omega(t-x/c)} + A(\omega)R(\omega)e^{i\omega(t+x/c)}$$

  – By knowing $p(t,x_j)$ at a number of location x_j an inverse problem can be set-up to determine the complex functions, $R(\omega)$ and $A(\omega)$

Module on Cryptography

- **Some elementary cryptosystems.**
- **The Hill cryptosystems.**
- **The Hill cryptosystem with Maple.**
- **Generalizations of the Hill cryptosystem.**
- **The two-message problem.**
- **Mathematical prerequisites for the RSA encryption**
- **The RSA encryption and decryption.**
- **The RSA cryptosystem and Maple.**

E. L. Stitzinger

Module on Modeling of Random Phenomena

- **Basic concepts in Probability. Bayes formula and decision examples.**
- **Bernoulli model. Examples of queuing systems. Markov chains and equilibrium.**
- **Random walks. A probabilistic approach to heat diffusion. Monte Carlo simulations.**
- **Law of Large Numbers. Example of a particle moving in a random medium.**
- **Central Limit Theorem. Estimation from large samples.**
- **Financial Mathematics. An introduction to the problem of pricing and hedging financial derivatives. The one-period model.**
- **Multi-period mode and binomial trees. Examples of American and exotic options.**
- **Black-Scholes formula from discrete to continuous time models. Brownian motion.**

Jean-Pierre Fouque

Module on Biological Modeling

- **Compartmental models. How is alcohol different from Prozac?**
- **Disease transmission. How does an epidemic happen? What happens when you vaccinate some of the population?**
- **Human population growth. When will Raleigh have 1 million people? How many people will the world have in the long run?**

S. R. Lubkin

Course Administration

- **Course coordinator**
  - Selects faculty
  - Introduces new modules
  - Reads end of semester papers
  - Tabulates course grades
- **Module instructors**
  - Prepare course material at the appropriate level
  - Grades some traditional homework
- **Initial work load credit:**
  - One three-credit course
  - equals
  - A single new module spread over three semesters

Present Status

- **Third year the course has been given**
- **Have had increasing enrollments….now 33**
- **Students from math, math education and prospective math majors**
- **Math elective**
- **Effective guidance tool**
- **Students mention it in exit interviews**