1. (a). \( x_h = Ce^{-3t} \).

(b). Assume \( x_p = a \cos(2t) + b \sin(2t) \)
\[
\begin{align*}
& a(-2) \sin(2t) + b \cos(2t) = -3(a \cos(2t) + b \sin(2t)) + \cos(2t) \\
& (-2a) \sin(2t) + 2b \cos(2t) = (-3b) \sin(2t) + (-3a + 1) \cos(2t).
\end{align*}
\]
This requires \(-2a = -3b\) and \(2b = -3a + 1\).
The algebraic solution is \( a = 3/13 \) and \( b = 2/13 \) giving
\[
x_p = (3/13) \cos(2t) + (2/13) \sin(2t).
\]
(c). The general solution is
\[
x(t) = Ce^{-3t} + (3/13) \cos(2t) + (2/13) \sin(2t).
\]
The initial condition gives
\[
x(0) = 2 = C + (3/13) + 0.
\]
Hence, \( C = 23/13 \) and
\[
x(t) = (23/13)e^{-3t} + (3/13) \cos(2t) + (2/13) \sin(2t).
\]

2. (a). \( r^2 + 16 = 0 \) gives \( r = 4i \) and \( r = -4i \).
The homogeneous solution is
\[
x_h = C_1 \cos(4t) + C_2 \sin(4t).
\]
(b). Compute the derivative
\[
x'_h = C_1 (-4) \sin(4t) + C_2 \cos(4t).
\]
The initial condition requires
\[
x_h(0) = 2 = C_1 + 0 \quad \text{and} \quad x'_h(0) = 1 = 0 + C_2 \cos(0).
\]
Thus, \( C_1 = 2, C_2 = 1/4 \) and the solution is
\[
x_h = 2 \cos(4t) + (1/4) \sin(4t).
\]

3. (a). \( r^2 - 9 = 0 \) gives \( r = 3 \) and \( r = -3 \).
The homogeneous solution is
\[
x_h = C_1 e^{3t} + C_2 e^{-3t}.
\]
(b). Assume \( x_p = at^2 + bt + c \)
\[
(2a) - 9(at^2 + bt + c) = 9t^2 + 18t + 5
\]
\[
t^2(-9a) + t(-9b) + (2a - 9c) = 9t^2 + 18t + 5.
\]
This requires \(-9a = 9, -9b = 18 \) and \(2a - 9c = 5\).
Thus, \( a = -1, b = -2 \) and \( c = -7/9 \) giving
\[
x_p = -t^2 - 2t - 7/9.
\]
(c). The general solution is
\[
x(t) = C_1 e^{3t} + C_2 e^{-3t} - t^2 - 2t - 7/9.
\]
Compute the derivative
\[
x'(t) = C_1 3e^{3t} + C_2 (-3)e^{-3t} - 2t - 2.
\]
The initial conditions require
\[
x(0) = 0 = C_1 + C_2 - 7/9 \quad \text{and} \quad x'(0) = 0 = C_1 3 + C_2 (-3) - 2.
\]
The solution of the algebraic problem is
\[
C_1 = 13/18 \quad \text{and} \quad C_2 = 1/18 \quad \text{giving}
\]
\[
x(t) = (13/18)e^{3t} + (1/18)e^{-3t} - t^2 - 2t - 7/9.
\]

4. Let \( \det(A - rI) = 0 \)
\[
\det\left[ \begin{array}{cc}
3 - r & 7 \\
1 & -3 - r
\end{array} \right] = 0
\]
\[
r^2 - 16 = 0
\]
\[
r = 4 : \quad \left[ \begin{array}{cc}
3 - 4 & 7 \\
1 & -3 - 4
\end{array} \right] \left[ \begin{array}{c}
u_1 \\
u_2
\end{array} \right] = \left[ \begin{array}{c}
0 \\
0
\end{array} \right]
\]
\[-u_1 + 7u_2 = 0 \quad \text{and} \quad u_1 - 7u_2 = 0.
\]
Set \( u_2 = 1 \) giving \( u_1 = 7 \) and the
5. The general homogeneous solution is
   \[
   \begin{bmatrix}
   x(t) \\
y(t)
   \end{bmatrix} = C_1 \begin{bmatrix} 7 \\ 1 \end{bmatrix} e^{4t} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-4t}.
   \]

   The initial conditions give
   \[
   \begin{bmatrix}
   x(0) \\
y(0)
   \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = C_1 \begin{bmatrix} 7 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}.
   \]

   This algebraic system may be written as
   \[
   \begin{bmatrix}
   7 & -1 \\
1 & 1
   \end{bmatrix}
   \begin{bmatrix}
   C_1 \\
C_2
   \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.
   \]

   The algebraic solution is
   \[
   C_1 = \det(\begin{bmatrix}
   1 & -1 \\
1 & 1
   \end{bmatrix})/8 = 2/8 \quad \text{and} \quad
   C_2 = \det(\begin{bmatrix}
   7 & 1 \\
1 & 1
   \end{bmatrix})/8 = 6/8.
   \]

   The solution of the initial value problem is
   \[
   \begin{bmatrix}
   x(t) \\
y(t)
   \end{bmatrix} = (1/4) \begin{bmatrix} 7 \\ 1 \end{bmatrix} e^{4t} + (3/4) \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-4t}.
   \]