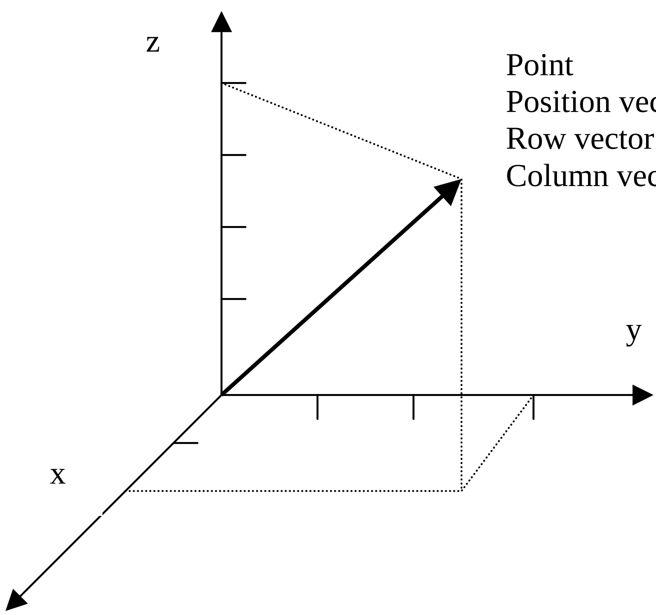
Lecture 2: Curves in Space and Projectiles

In this lecture we will use column vectors to plot a sequence of points in space to depict a curve in space. The coordinates of the points will be stored in either row vectors or column vectors. Application to the path of a projectile with air resistance will be given, and a Matlab demo proj3d.m will use the Matlab command plot3 to generate a graph.

Points in space are usually given by ordered triples (x ,y ,z). One could view this as either a 3x1 column vector, or a 1x3 row vector, or as a "position vector" starting at the point (0, 0, 0) and ending at the point (x, y, z). This does create some confusion, but the literature is laden these different perspectives.

Example. Graph the point (2, 3, 4).

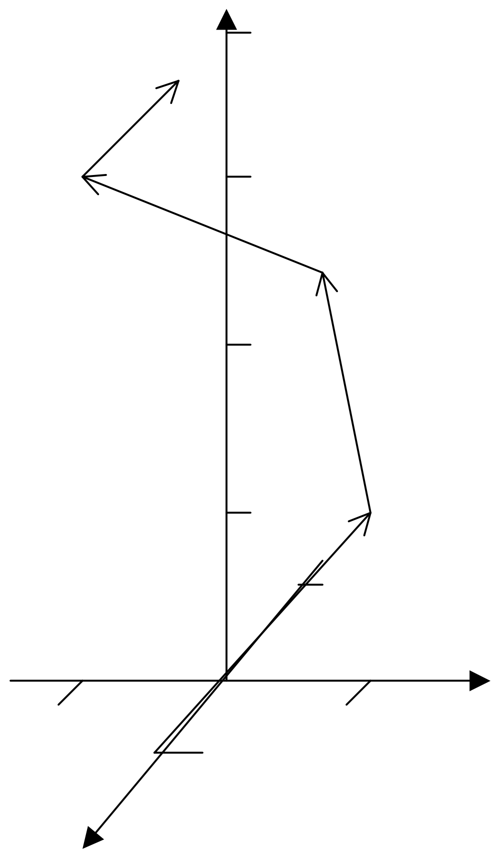
Point = (2 , 3, 4)
Position vector = 2 **i** + 3 **j** + 4 **k**
Row vector = [2 3 4]
Column vector = $[2\ 3\ 4]^T$



If we were to graph a curve in space by hand, we would form a table with four columns of numbers. The first column would be the possible times, t_i where i = 1,..,n. The second, third and fourth columns would contain the computed coordinates $x(t_i)$, $y(t_i)$ and $z(t_i)$. Then each of the points would be plotted. If there were enough points (n is suitably large), then the points could be connected by a straight line to form a depiction of the curve.

Example. A helix is the set of points given by (cos(t), sin(t), t), that is, x(t) = cos(t), y(t) = sin(t) and z(t) = t. Since $x^2 + y^2 = \cos^2(t) + \sin^2(t) = 1$, the curve must be in the vertical cylinder of radius one. You may want to view this as a path of a handrail on a spiral staircase. The following table has only five points, and it could be viewed as a 5x4 matrix.

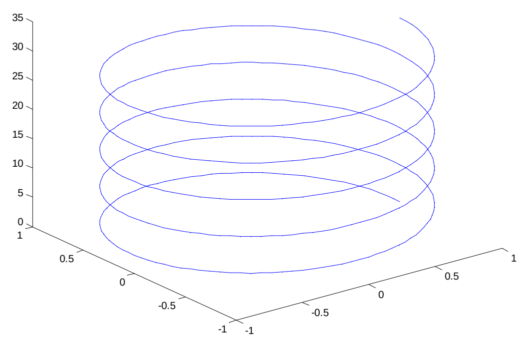
t	**x(t)**	**y(t)**	**z(t)**
0	1	0	0
$\pi/2$	0	1	$\pi/2$
π	-1	0	π
$3\pi/2$	0	-1	$3\pi/2$
2π	1	0	2π



In order to obtain a more accurate depiction of the helix, one needs many more points. The following Matlab commands generate four 501x1 row vectors t, x , y and z:

```
t = 0:pi/50:10*pi;
x = cos(t);
y = sin(t);
z = t'
plot3(x,y,z)
```

The first line generates t from 0 to 10*pi in increments equal to pi/50. The second, third and fourth lines use array operations to compute x, y and z at each of the time values. The last line uses the Matlab command plot3 to generate the following depiction of the helix



Application to a Projectile in Space. Consider a projectile whose initial position is (0, 0 ,0) and initial velocity (x'(0), y'(0), z'(0)) is given. Suppose the forces are from gravitation and air resistance. The air resistance will be assumed to be proportional to the speed in each direction; this may not be true if the speed is too large or the elevation is too high. Let c be the proportionality constant. Assume there is a non-zero wind vector W = [b a 0]. Newton's Law of motion applied to each of the three directions is as follows:

$mx'' = -c\,b - c\,x'$,

$my'' = -c\,a - c\,y'$ and

$mz'' = -mg - c\,z'$.

These are all of the form mv' = d – c v. Assume m = 1 so that the solution can be found via the transformation w = d –c v as follows:

$$w' = (d - c\,v)'$$

$$\quad = 0 - c\,v'$$

$$v' = w'/c = w$$

$$w' = -c\,w$$

$$w(t) = w(0)\,e^{-ct}$$

$$d - c\,v(t) = (d - c\,v(0))\,e^{-ct}$$

$$v(t) = d/c - (d/c - v(0))\,e^{-ct}.$$

Apply this to v = x' and y' and z', and then integrate each of these functions to find x(t) , y(t) and z(t).

In the Matlab demo proj3d.m the following constants are used:

mass = m = 1

gravitation = g = 32

wind speed in the y direction = a = -10

wind speed in the x direction = b = 5

wind speed in the z direction = 0

resistance coefficient = c = .01

initial speed in the x direction = 0

initial speed in the y direction = 500 and

initial speed in the z direction = 500.

The reader will find it interesting to experiment with these parameters in proj3d.m.

Homework.

1. By hand graph the helix for t = 0:pi/4:2*pi, that is, use 9 data points.
2. Use the Matlab command plot3 to graph the helix. Graph the spiral outward variation of the helix x = t cos(t), y = t sin(t) and z = t.
3. In the projectile problem find x(t), y(t) and z(t).
4. In proj3d.m experiment with different wind and initial velocities as well as different air resistance coefficients.