Lecture 1: Matrices and Plotting Inventory

Matrices, often called arrays, are used to store information. Here the information will be numbers, which will refer to quantities, proportionality constants, or rates of change. In the first three lectures the matrices will store information that is relevant to different graphs. The first lecture will have graphs in 2D, the second lecture will plot curves in 3D, and the third will graph surfaces in 3D. Applications will be given to inventory versus time, coordinates of projectile as a function of time, and a cost function of two variables. Computer implementations will be given in Matlab. The demonstrations can be run by simply typing, at the Matlab prompt, the name of the demo and hitting any key to advance the demo.

In order to form a graph one usually starts with a table of numbers with two columns and n data points. For example, consider the inventory of TV sets at the end of each month.

Months	**Number of TVs**
1 (Jan.)	40
2 (Feb.)	35
3 (Mar.)	30
4 (Apr.)	52
5 (May)	22
6 (June)	25
7 (July)	26
8 (Aug.)	27
9 (Sept.)	20
10 (Oct.)	37
11 (Nov.)	50
12 (Dec.)	60

One can view this as two columns each with 12 entries, or as a 12x2 matrix with 12 rows and 2 columns. If the inventory is doubled, then each entry in the second column is multiplied by 2. If there is a second store, then the total inventory is a column vector formed by adding the entries for each month.

Definition. A = *nx1 column* vector has n entries listed in a single column.

$$A = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = [a_i] \; with \; i = 1, \cdots, n$$

Definition. The following are four basic operations on a column vector.

Let c be a number, A and B be nx1 column vectors.

Scalar multiplication: cA is another nx1 column vector whose entries are c times the entries of A, that is,

$$cA = \begin{bmatrix} ca_1 \\ \vdots \\ ca_n \end{bmatrix} = [ca_i] \; with \; i = 1, \cdots, n.$$

Vector Addition: A+B is another nx1 column vector whose entries are the sum of the entries of A and B, that is,

$$A + B = \begin{bmatrix} a_1 + b_1 \\ \vdots \\ a_n + b_n \end{bmatrix} = [a_i + b_i] \; with \; i = 1, \cdots, n$$

Augmentation of A and B: [A B] is a nx2 matrix whose first column is A and second column is B, that is,

$$\begin{bmatrix} A & B \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ \vdots & \vdots \\ a_n & b_n \end{bmatrix} = [a_i \;\; b_i] \; with \; i = 1, \cdots, n$$

Transpose of A: A^T is a 1xn row vector with the same entries of A listed as a row, that is,

$$A^T = [a_1 \cdots a_n].$$

Examples. Let Stores A and B have the following inventories over the last six months of the year so that A and B will both be 6x1 column vectors.

$$A = \begin{bmatrix} 26 \\ 27 \\ 20 \\ 37 \\ 50 \\ 60 \end{bmatrix} \; and \; B = \begin{bmatrix} 16 \\ 17 \\ 10 \\ 27 \\ 30 \\ 40 \end{bmatrix}$$

If store A doubles its inventory and store B triples its inventory, then the total inventory will be 2A + 3B and the augmented inventory matrix will be [2A 3B]

$$2A + 3B = \begin{bmatrix} 52+48 \\ 54+51 \\ 40+30 \\ 74+81 \\ 100+90 \\ 120+120 \end{bmatrix} = \begin{bmatrix} 100 \\ 105 \\ 70 \\ 155 \\ 190 \\ 240 \end{bmatrix} \; and$$

$$\begin{bmatrix} 2A & 3B \end{bmatrix} = \begin{bmatrix} 52 & 48 \\ 54 & 51 \\ 40 & 30 \\ 74 & 81 \\ 100 & 90 \\ 120 & 120 \end{bmatrix}$$

In Matlab matrices are initialized exactly as above (see the Matlab demo inventory.m). Once the matrices have been initialized, then the Matlab command plot can be used to easily generate a graph. For example, if months is the transpose of [7 8 9 10 11 12], then the Matlab command plot(months, A) will generate a line graph of store A inventory versus months.

Not all inventories are functions of discrete time. Suppose a chemical, say salt, is uniformly mixed into water in a 1000 liter tank. Suppose there is an incoming solution with concentration equal to 1 kg/liter, and flow rate equal to 6 liter/min. If the tank remains full, then the outgoing concentration is x(t)/1000 kg/liter where x(t) is the salt (inventory) in the tank at time t, and the flow rate out must also be 6 liter/min. This means x(t) must satisfy the differential equation $x' = 6 - 6x/1000$. If the $x(0) = 0$, then one can show, via $z = 6 - 6x/1000$ or dsolve,

$$x(t) = 1000 - 1000\, e^{(-6/1000)\, t}.$$

In order to graph this, one needs to form a table to times, t, and values of x(t). In Matlab we need two columns or two row vectors and to use of the command plot. See inventory.m where

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t = 0 :.5:100;
x = 1000 - 1000*exp((-6/100)*t);
plot(t,x)
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The first line generates a row vector with 201 entries from 0 to 100 in increments equal to .5. The second line also generates a row vector with 201 entries with values equal to 1000 - 1000*exp((-6/100)*t_i) where t_i are from line one.

This is an example of an *array operation*, and other array operations include:

Let $A = [a_i]$ and $B = [b_i]$ be arrays, say nx1,

$f(A) = [f(a_i)]$ is a nx1 array,

A .*B = $[a_i\, b_i]$ is a nx1 array,

A ./B = $[a_i / b_i]$ is a nx1 array and

A .^n = $[(a_i)^n]$ is a nx1 array.

Homework.

1. Let $A = [\ 3\ 5\ 7\ {-1}]^T$, $B = [-1\ 2\ 5\ 11]^T$ and c =7.
   Find the following: A + B, cA, [A 2B], exp(A) and A .^2.
2. In problem one verify c(A + B) = cA + cB. Is this true for all nx1 row vectors? Justify your answer.
3. Use Matlab to do problem 1.
4. Use Matlab to graph $e^{-t}\sin(t)$ when t varies from 0 to 6.