Lecture 28

Variational Inequalities: porous flow

(Lecture notes taken by ????? and ???)

- Classical model for porous flow.
- Weak formulation.
- Baiocchi transformation.
- FEM discrete model.


\[
\begin{align*}
\text{(8.4) } & \quad \alpha = n \frac{\varepsilon}{\eta} \\
\text{(7.4) } & \quad \beta = n \\
\text{(9.4) } & \quad \gamma = n \\
\text{(5.4) } & \quad \delta = n \\
\end{align*}
\]

where \( n \) is the number of particles.

\[
\alpha = n \frac{\varepsilon}{\eta} + (\gamma + \delta - \eta)
\]

Classical Formulation of Fluid Flow in a porous medium

There are two fundamental approaches to the formulation of fluid flow in a porous medium: the Darcy's law and the Forchheimer equation. The Darcy's law is a linear relationship between the pressure drop and the flow rate, while the Forchheimer equation includes a quadratic term that accounts for the inertial effects.

\[
\text{(8.4) } \quad \alpha = n \frac{\varepsilon}{\eta} \\
\text{(7.4) } \quad \beta = n \\
\text{(9.4) } \quad \gamma = n \\
\text{(5.4) } \quad \delta = n
\]

\[
\text{Classical Formulation of the Water-Film Flow in a porous medium}
\]

where \( \eta \) is the water film thickness, \( \varepsilon \) is the porosity, \( \alpha \) is the Darcy's law coefficient, and \( \beta \) is the Forchheimer coefficient.
\[ \Delta_{J} \cap \Delta_{J} \cup \{0\} = (\Delta_{J})^{\infty} \quad (8) \]
\[ \Delta_{J} \cup \Delta_{J} \cup \{0\} = (\Delta_{J})^{\infty} \quad (9) \]
\[ \Delta_{J} \cup \{0\} = (\Delta_{J})^{\infty} \quad (10) \]
\[ \{0\} = (\Delta_{J})^{\infty} \quad (11) \]

\[ \text{Proposition 9.4.1:} \quad \text{Let } \varphi \text{ be a weak solution of } (1.4.6) \text{ on } \Omega \times (0, T) \text{ and } \varphi \neq 0. \]

\[ \text{The weak solution } \varphi \text{ is unique in } \Omega \times (0, T). \]

**Theorem:** (1.4.6) and (1.3.6)

\[ \text{Proposition 9.4.1:} \quad \text{Let } \varphi \text{ be a weak solution of } (1.4.6) \text{ on } \Omega \times (0, T) \text{ and } \varphi \neq 0. \]

\[ \sum_{i=1}^{n} \int_{\Omega} \varphi_{i} \varphi_{i} \cdot \nabla \varphi_{i} \text{ d}x = \int_{\Omega} \varphi \cdot \nabla \varphi \text{ d}x \]

\[ \text{Proposition 9.4.1:} \quad \text{Let } \varphi \text{ be a weak solution of } (1.4.6) \text{ on } \Omega \times (0, T) \text{ and } \varphi \neq 0. \]
\[ (x) \leq 1 \]
\[ (x) < 1 \]
\[ \int \psi(x) \, dx = 1 \]

The function \( \psi(x) \) is defined as \( \psi(x) = \exp(-x^2) \) for \( x \geq 0 \) and \( \psi(x) = 0 \) for \( x < 0 \).

**Problem:**

Let \( g(x) = xe^{-x^2} \) for \( x \geq 0 \) and \( g(x) = 0 \) for \( x < 0 \). Find the critical points of \( g(x) \).

**Solution:**

The critical points of \( g(x) \) occur where \( g'(x) = 0 \). We have

\[ g'(x) = e^{-x^2} - 2xe^{-x^2} = (1 - 2x)e^{-x^2} \]

Setting \( g'(x) = 0 \), we get \( x = 0.5 \). Therefore, the critical point is \( x = 0.5 \).

**Conclusion:**

The critical point of \( g(x) \) is \( x = 0.5 \).