

Separation of Variables and dsolve

**% Solution of $y' = c(\text{ysur} - y)$
and $y(0) = y_0$ via dsolve**

EDU» syms y c ysur y0 t

EDU» sol = dsolve('Dy = c*(ysur - y)', 'y(0) = y0')

sol =

ysur+exp(-c*t)*(-ysur+y0)

EDU» sol = subs(sol, {ysur,c},{70,.01})

sol =

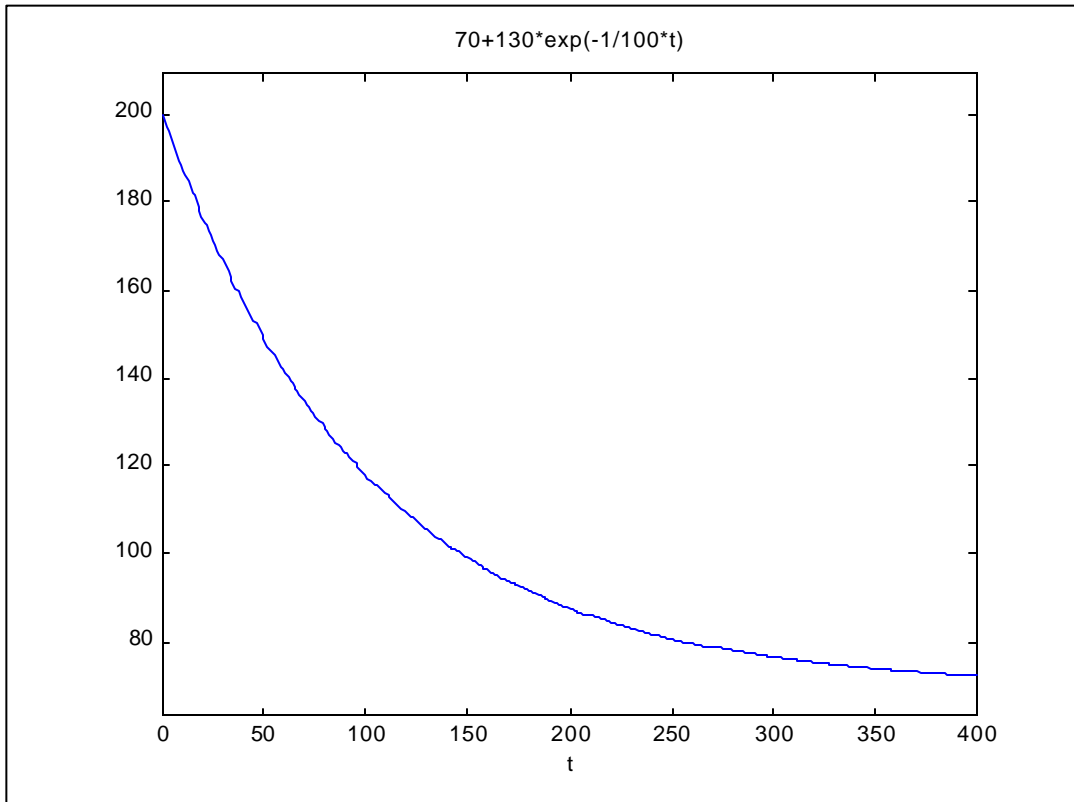
70+exp(-1/100*t)*(-70+y0)

EDU» sol200 = subs(sol, y0,200)

sol200 =

70+130*exp(-1/100*t)

EDU» ezplot(sol200, [0 400])



**% Solution of $y' = c(y - 1)(y - 8)$
and $y(0) = y_0$**

% First, use dsolve.

EDU» sol = dsolve('Dy = c*(y-1)*(y-8)', 'y(0)= y0')

sol =

$(-8 + \exp(7*c*t - \log((-1+y_0)/(-8+y_0)))) / (\exp(7*c*t - \log((-1+y_0)/(-8+y_0))) - 1)$

EDU» syms c y0 y t

EDU» sol1 = subs(sol,c,.01)

sol1 =

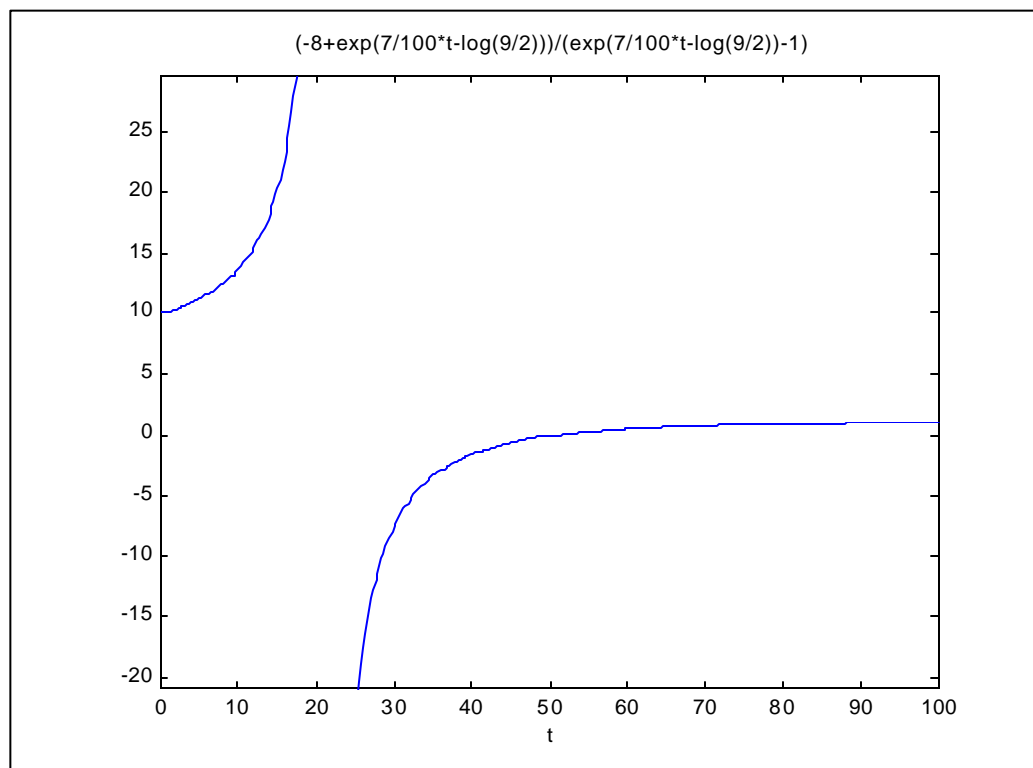
$(-8 + \exp(7/100*t - \log((-1+y_0)/(-8+y_0)))) / (\exp(7/100*t - \log((-1+y_0)/(-8+y_0))) - 1)$

EDU» sol10 = subs(sol1, y0 , 10)

sol10 =

$(-8+\exp(7/100*t-\log(9/2)))/(\exp(7/100*t-\log(9/2))-1)$

EDU» ezplot(sol10 , [0 100])

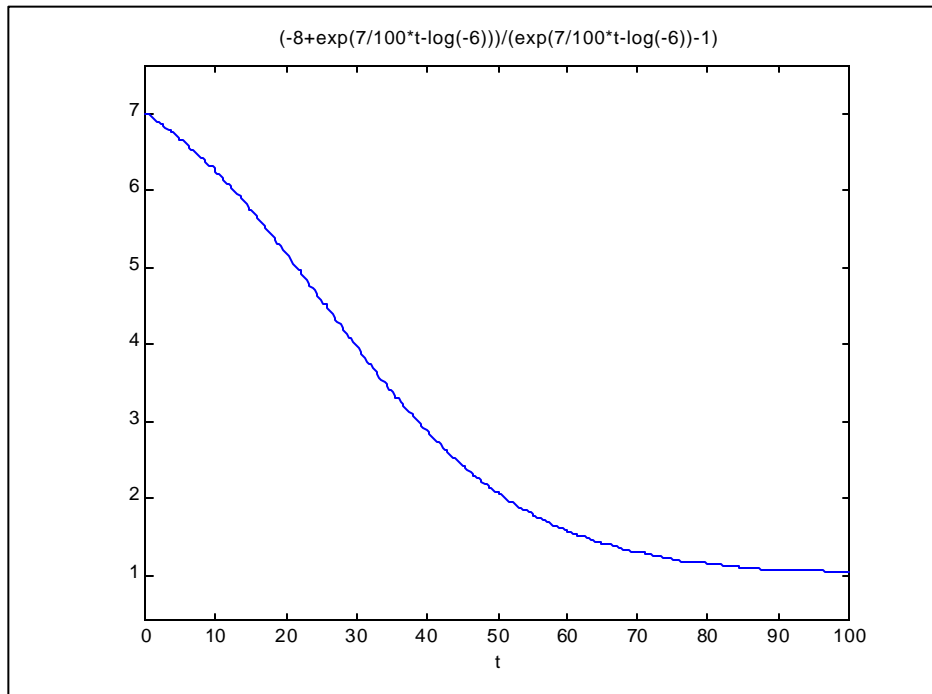


EDU» sol7 = subs(sol1,y0, 7)

sol7 =

$(-8+\exp(7/100*t-\log(-6)))/(\exp(7/100*t-\log(-6))-1)$

EDU» ezplot(sol7,[0 100])

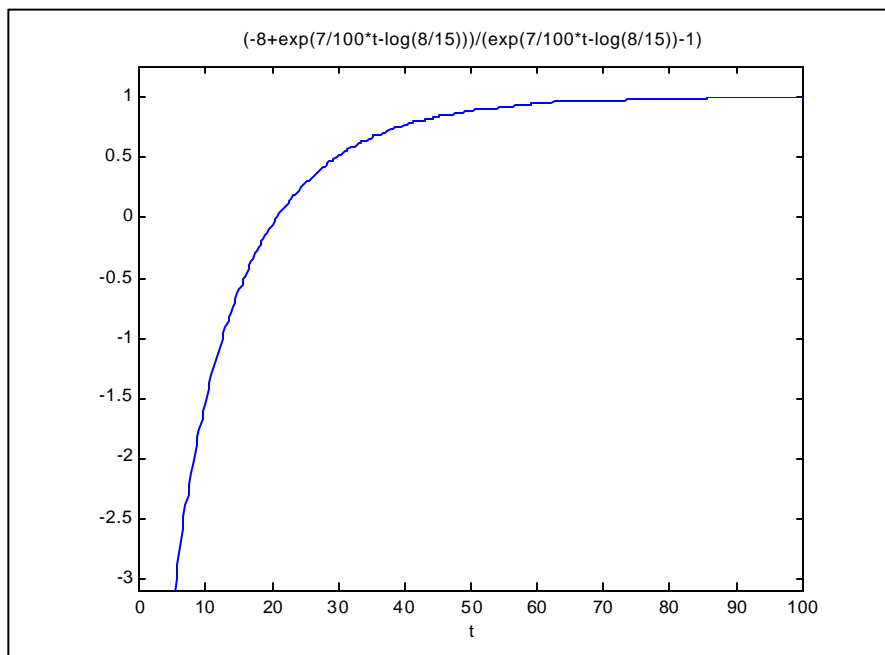


EDU» solm7 = subs(sol1,y0,-7)

solm7 =

$(-8+\exp(7/100*t-\log(8/15)))/(\exp(7/100*t-\log(8/15))-1)$

EDU» ezplot(solm7,[0 100])



**% Solution of $y' = c(y - 1)(y - 8)$
and $y(0) = y_0$**

% Second, use separation of variables.

EDU» clear

EDU» syms left right y t sol y0

EDU» left = int('1/((y-1)*(y-8))','y')

left = -1/7*log(y-1)+1/7*log(y-8)

EDU» right = int('c','t')

right = c*t

EDU» lefttop = subs(left,y,sol)

lefttop = -1/7*log(sol-1)+1/7*log(sol-8)

EDU» leftbottom = subs(left,y,y0)

leftbottom = -1/7*log(-1+y0)+1/7*log(-8+y0)

EDU» solve(' -1/7*log(sol-1)+1/7*log(sol-8)-(-1/7*log(-1+y0)+1/7*log(-8+y0))=c*t', 'sol')

$$\text{ans} = \frac{(-8*\exp(7*c*t)+\exp(7*c*t)*y0+8-8*y0)}{(-8*\exp(7*c*t)+\exp(7*c*t)*y0+1-y0)}$$

EDU» dsolve('Dy=(y-1)*(y-8)', 'y(0)=y0')

$$\text{ans} = \frac{(-8+\exp(7*t-\log((-1+y0)/(-8+y0))))}{(\exp(7*t-\log((-1+y0)/(-8+y0)))-1)}$$

EDU» simplify(ans)

$$\text{ans} = \frac{(-8*\exp(7*c*t)+\exp(7*c*t)*y0+8-8*y0)}{(-8*\exp(7*c*t)+\exp(7*c*t)*y0+1-y0)}$$