

## Extra Credit 2 in MA 341.002

**Due Date is Tuesday, October 5.**

**How much extra credit? Up to 10 percent on exam two.**

Consider the initial value problem for a mass-spring system with mass equal to 4, damping equal to 4, spring constant equal to 3, initial position is  $-1$  and initial velocity is 1. Consult the Matlab/Maple links for lectures 10, 20 and 21 in the old syllabus:

<http://www4.eos/users/w/white/www/white/ma341/ma341hp.htm>

- (a). By hand calculations and the method of undetermined coefficients find the solution when the external force is  $2 + t - t^2 + 4t^3$ .
- (b). Use *dsolve* and *ezplot* to find this solution and its graph.
- (c). Consider the above problem when the external force is  $10\sin(4t)$  and the damping constant is 4, 5 and 6. Use *dsolve* and *ezplot* to find the three solutions and compare the graphs.
- (d). Consider the above problem when the external force is  $10\sin(4t)$  and the damping constant is 6 and the spring constant is 1, 2 and 3. Use *dsolve* and *ezplot* to find the three solutions and compare the graphs.

I a

$$4y'' + 4y' + 3y = 2 + t - t^2 + 4t^3, \quad y(0) = -1$$

$$y'(0) = 1$$

hom. sol:  $4y'' + 4y' + 3y = 0$

$$r = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 4 \cdot 3}}{2 \cdot 4} = \frac{-4 \pm i4\sqrt{2}}{8} = -\frac{1}{2} \pm i \frac{\sqrt{2}}{2}$$

$$y_1 = e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{2}}{2}t\right), \quad y_2 = e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{2}}{2}t\right)$$

part. sol.:

$$y_p = A_0 + A_1 t + A_2 t^2 + A_3 t^3$$

$$y_p' = 0 + A_1 + 2A_2 t + 3A_3 t^2$$

$$y_p'' = 0 + 0 + 2A_2 + 6A_3 t$$

$$4(2A_2 + 6A_3 t) + 4(A_1 + 2A_2 t + 3A_3 t^2) + 3(A_0 + A_1 t + A_2 t^2 + A_3 t^3) = 2 + t - t^2 + 4t^3$$

$$t^3 (3A_3)$$

$$+ t^2 (3A_2 + 12A_3)$$

$$+ t (3A_1 + 8A_2 + 3A_3)$$

$$+ (8A_2 + 4A_1 + 3A_0)$$

$$So, \quad 3A_3 = 4 \quad \rightarrow \quad A_3 = \frac{4}{3}$$

$$3A_2 + 12A_3 = -1 \quad \rightarrow \quad A_2 = \frac{(-1 - 12(\frac{4}{3}))}{3} = -\frac{17}{3}$$

$$3A_1 + 8A_2 + 3A_3 = 1 \quad \rightarrow \quad A_1 = \frac{(1 - 8(\frac{17}{3}) - 3(\frac{4}{3}))}{3} = \frac{43}{9}$$

$$8A_2 + 4A_1 + 3A_0 = 2 \quad \rightarrow \quad A_0 = \frac{(2 - 4(\frac{43}{9}) - 8(\frac{17}{3}))}{3} = \frac{254}{27}$$

gen. sol:

$$y = C_1 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{2}}{2}t\right) + C_2 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{2}}{2}t\right) + \left(\frac{254}{27} + \frac{43}{9}t - \frac{17}{3}t^2 + \frac{4}{3}t^3\right)$$

find  $C_1$  &  $C_2$ :

$$y(0) = -1 = C_1 \cdot 1 + C_2 \cdot 0 + \frac{254}{27} \quad \rightarrow \quad C_1 = -1 - \frac{254}{27} = \frac{-281}{27}$$

$$y'(t) = \left( -\frac{1}{2} C_1 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{2}}{2}t\right) - \frac{\sqrt{2}}{2} C_1 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{2}}{2}t\right) + \left(-\frac{1}{2} C_2 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{2}}{2}t\right) + \frac{\sqrt{2}}{2} C_2 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{2}}{2}t\right)\right) \right)$$

$$+ C_2 \left( e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{2}}{2}t\right) - \frac{\sqrt{2}}{2} C_2 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{2}}{2}t\right) \right)$$

$$+ 0 + \frac{43}{9} - \frac{17}{3} \cdot 2t + \frac{4}{3} \cdot 4t^2$$

$$y'(0) = 1 = C_1 \left(0 - \frac{1}{2}\right)$$

$$+ C_2 \left(\frac{\sqrt{2}}{2} + 0\right)$$

$$1 = \frac{-281}{27} \left(-\frac{1}{2}\right) + C_2 \frac{\sqrt{2}}{2} + \frac{43}{9} \quad \rightarrow \quad C_2 = \frac{\left(1 - \frac{43}{9} - \frac{281}{27} \cdot \frac{1}{2}\right) \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$= \frac{(54 - 258 - 281)}{54} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-485}{54}$$

$$y(t) = \left( \frac{-281}{27} e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{2}}{2}t\right) + \left(\frac{485}{54} \sqrt{2}\right) e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{2}}{2}t\right) + \frac{254}{27} + \frac{43}{9}t - \frac{17}{3}t^2 + \frac{4}{3}t^3 \right)$$

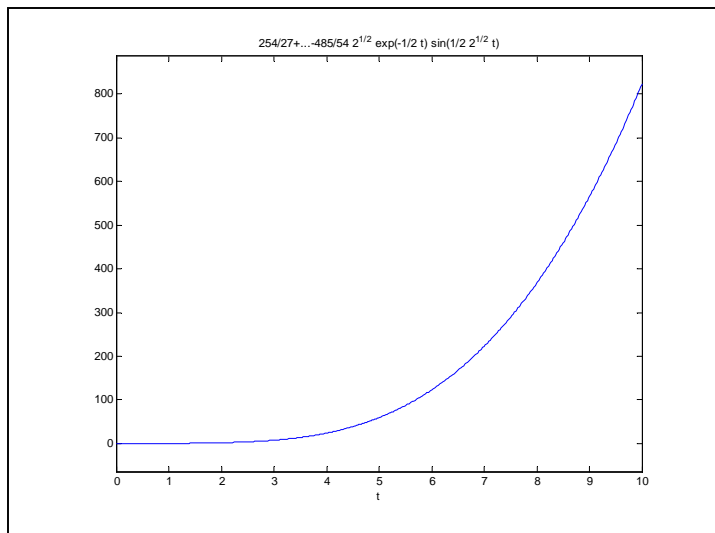
## 1b. Solution using dsolve.

```
EDU>> dsolve('4*D2y+4*Dy+3*y = 2+t-t^2+4*t^3','y(0) = -1','Dy(0)=1','t')
```

ans =

```
254/27+43/9*t-17/3*t^2+4/3*t^3-281/27*exp(-1/2*t)*cos(1/2*2^(1/2)*t)-  
485/54*2^(1/2)*exp(-1/2*t)*sin(1/2*2^(1/2)*t)
```

```
EDU>> ezplot(ans,[0 10])
```



**Remark.** The symbolic solution from various applications or versions of the same application may generate different representations of the symbolic solution. The Matlab command *simple(symbolic expression)* is combination of other commands such as *pretty*, *collect*, *expand* and *simplify*. As indicated in the following symbolic calculations this greatly reduces the symbolic output so that one can easily identify the homogeneous and particular parts of the general solution.

**1c. Solution of  $4y'' + by' + 3y = 10\sin(4t)$   
vary  $b = 4, 5$  and  $6$ .**

EDU>> solb4 = dsolve('4\*D2y+4\*Dy+3\*y = 10\*sin(4\*t)', 'y(0) = -1', 'Dy(0)=1', 't')

solb4 =

305/3977\*sin(1/2\*2^(1/2)\*t)\*cos(1/2\*(2^(1/2)+8)\*t)-  
80/3977\*cos(1/2\*2^(1/2)\*t)\*cos(1/2\*(2^(1/2)-  
8)\*t)+305/3977\*cos(1/2\*2^(1/2)\*t)\*sin(1/2\*(2^(1/2)-8)\*t)-  
80/3977\*cos(1/2\*2^(1/2)\*t)\*cos(1/2\*(2^(1/2)+8)\*t)-  
305/3977\*cos(1/2\*2^(1/2)\*t)\*sin(1/2\*(2^(1/2)+8)\*t)-  
80/3977\*sin(1/2\*2^(1/2)\*t)\*sin(1/2\*(2^(1/2)+8)\*t)-  
305/3977\*sin(1/2\*2^(1/2)\*t)\*cos(1/2\*(2^(1/2)-8)\*t)-  
80/3977\*sin(1/2\*2^(1/2)\*t)\*sin(1/2\*(2^(1/2)-8)\*t)-  
335/7954\*2^(1/2)\*cos(1/2\*2^(1/2)\*t)\*cos(1/2\*(2^(1/2)-  
8)\*t)+1260/3977\*2^(1/2)\*cos(1/2\*2^(1/2)\*t)\*sin(1/2\*(2^(1/2)-  
8)\*t)+335/7954\*2^(1/2)\*cos(1/2\*2^(1/2)\*t)\*cos(1/2\*(2^(1/2)+8)\*t)+1260/3977\*2^(1/2)  
\*cos(1/2\*2^(1/2)\*t)\*sin(1/2\*(2^(1/2)+8)\*t)-  
1260/3977\*2^(1/2)\*sin(1/2\*2^(1/2)\*t)\*cos(1/2\*(2^(1/2)+8)\*t)+335/7954\*2^(1/2)\*sin(1/  
2\*2^(1/2)\*t)\*sin(1/2\*(2^(1/2)+8)\*t)-  
1260/3977\*2^(1/2)\*sin(1/2\*2^(1/2)\*t)\*cos(1/2\*(2^(1/2)-8)\*t)-  
335/7954\*2^(1/2)\*sin(1/2\*2^(1/2)\*t)\*sin(1/2\*(2^(1/2)-8)\*t)-3817/3977\*exp(-  
1/2\*t)\*cos(1/2\*2^(1/2)\*t)+9017/7954\*2^(1/2)\*exp(-1/2\*t)\*sin(1/2\*2^(1/2)\*t)

EDU>> solb4 = simple(solb4)

solb4 =

-610/3977\*sin(4\*t)-160/3977\*cos(4\*t)-3817/3977\*exp(-  
1/2\*t)\*cos(1/2\*2^(1/2)\*t)+9017/7954\*exp(-1/2\*t)\*sin(1/2\*2^(1/2)\*t)\*2^(1/2)

EDU>> ezplot(solb4, [0 10])

EDU>> hold on

EDU>> solb5 = dsolve('4\*D2y +5\*Dy + 3\*y= 10\*sin(4\*t)', 'y(0)=-1', 'Dy(0)=1'); solb5 =  
simple(solb5)

solb5 =

-610/4121\*sin(4\*t)-200/4121\*cos(4\*t)-3921/4121\*exp(-  
5/8\*t)\*cos(1/8\*23^(1/2)\*t)+32883/94783\*exp(-5/8\*t)\*sin(1/8\*23^(1/2)\*t)\*23^(1/2)

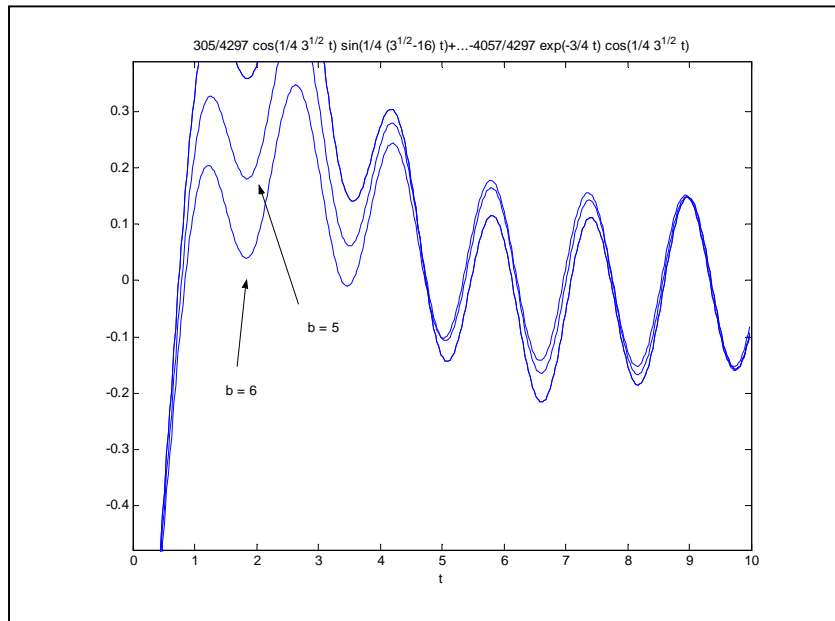
EDU>> ezplot(solb5, [0 10])

```
EDU>> solb6 = dsolve('4*D2y +6*Dy + 3*y= 10*sin(4*t)', 'y(0)=-1', 'Dy(0)=1'); solb6 = simple(solb6)
```

solb6 =

$$-610/4297*\sin(4*t)-240/4297*\cos(4*t)-4057/4297*\exp(-3/4*t)*\cos(1/4*3^{1/2}*t)+14777/12891*\exp(-3/4*t)*\sin(1/4*3^{1/2}*t)*3^{1/2}$$

```
EDU>> ezplot(solb6, [0 10])
```



## 1d. Solution of $4y'' + 4y' + ky = 10\sin(4t)$ vary $k = 1, 2$ and $3$ .

```
EDU>> solk1 = dsolve('4*D2y +4*Dy + 1*y= 10*sin(4*t)', 'y(0)=-1', 'Dy(0)=1'); solk1 = simple(solk1)
```

solk1 =

$$-32/845*\cos(4*t)-126/845*\sin(4*t)-813/845*\exp(-1/2*t)+29/26*\exp(-1/2*t)*t$$

```
EDU>> ezplot(solk1, [0 10])
```

```
EDU>> hold on
```

```
EDU>> solk2 = dsolve('4*D2y +4*Dy + 2*y= 10*sin(4*t)', 'y(0)=-1', 'Dy(0)=1'); solk2 = simple(solk2)
```

solk2 =

$$-8/205*\cos(4*t)-31/205*\sin(4*t)-197/205*\exp(-1/2*t)*\cos(1/2*t)+461/205*\exp(-1/2*t)*\sin(1/2*t)$$

```
EDU>> ezplot(solk2, [0 10])
```

```
EDU>> solk3 = dsolve('4*D2y +4*Dy + 3*y= 10*sin(4*t)', 'y(0)=-1', 'Dy(0)=1'); solk3 = simple(solk3)
```

solk3 =

$$-610/3977*\sin(4*t)-160/3977*\cos(4*t)-3817/3977*\exp(-1/2*t)*\cos(1/2*2^{1/2}*t)+9017/7954*\exp(-1/2*t)*\sin(1/2*2^{1/2}*t)*2^{1/2}$$

```
EDU>> ezplot(solk3, [0 10])
```

