Extra Credit 2 in MA 341.002

Due Date is Tuesday, October 5.
How much extra credit? Up to 10 percent on exam two.

Consider the initial value problem for a mass-spring system with mass equal to 4, damping equal to 4, spring constant equal to 3, initial position is –1 and initial velocity is 1. Consult the Matlab/Maple links for lectures 10, 20 and 21 in the old syllabus:

http://www4/eos/users/w/white/www/white/ma341/ma341hp.htm

(a). By hand calculations and the method of undetermined coefficients find the solution when the external force is \(2 + t - t^2 + 4t^3\).

(b). Use \texttt{dsolve} and \texttt{ezplot} to find this solution and its graph.

(c). Consider the above problem when the external force is \(10\sin(4t)\) and the damping constant is 4, 5 and 6. Use \texttt{dsolve} and \texttt{ezplot} to find the three solutions and compare the graphs.

(d). Consider the above problem when the external force is \(10\sin(4t)\) and the damping constant is 6 and the spring constant is 1, 2 and 3. Use \texttt{dsolve} and \texttt{ezplot} to find the three solutions and compare the graphs.
\[ 4y'' + 4y' + 3y = 2 + e^{-2x} y^2. \]

**Initial Conditions:**
\[ y(0) = -1, \quad y'(0) = 1. \]

**Homogeneous Solution:**
\[ \gamma_1 = e^{-3x}, \quad \gamma_2 = e^{-x}. \]

**Particular Solution:**
\[
\begin{align*}
\gamma_3 &= A_0 + A_1 x + A_2 x^2 + A_3 x^3 \\
A_0 &= 1, \quad A_1 = -2, \quad A_2 = 3, \quad A_3 = 4 \\
\gamma_3' &= 3 A_2 x + 6 A_3 x^2 + 3 A_3 x^3 \\
\gamma_3'' &= 6 A_3 + 6 A_3 x \\
y(x) &= \gamma_1 + \gamma_2 + \gamma_3 \\
\end{align*}
\]

**System:**
\[
\begin{align*}
A_0 &= y' \\
3 A_0 + 2 A_2 &= -1 \\
3 A_1 + 2 A_2 + 2 A_3 &= 1 \\
A_0 + 4 A_2 + 5 A_3 &= 2 \\
\end{align*}
\]

**General Solution:**
\[
y = c_1 e^{\frac{1}{3} x} + c_2 e^{\frac{1}{3} x} \sin\left(\frac{\sqrt{2}}{3} x\right) + c_3 e^{\frac{1}{3} x} \cos\left(\frac{\sqrt{2}}{3} x\right) \\
\]

**Initial Conditions:**
\[ y(0) = -1, \quad y'(0) = 1. \]

**Solutions:**
\[
\begin{align*}
y(x) &= c_1 e^{\frac{1}{3} x} + c_2 e^{\frac{1}{3} x} \sin\left(\frac{\sqrt{2}}{3} x\right) + c_3 e^{\frac{1}{3} x} \cos\left(\frac{\sqrt{2}}{3} x\right) \\
\end{align*}
\]
1b. Solution using dsolve.

EDU>> dsolve('4*D2y+4*Dy+3*y = 2+t-t^2+4*t^3','y(0) = -1','Dy(0)=1','t')
ans =
254/27+43/9*t-17/3*t^2+4/3*t^3-281/27*exp(-1/2*t)*cos(1/2*2^(1/2)*t)-
485/54*2^(1/2)*exp(-1/2*t)*sin(1/2*2^(1/2)*t)
EDU>> ezplot(ans,[0 10])

Remark. The symbolic solution from various applications or versions of the same application may generate different representations of the symbolic solution. The Matlab command `simple(symbolic expression)` is combination of other commands such as pretty, collect, expand and simplify. As indicated in the following symbolic calculations this greatly reduces the symbolic output so that one can easily identify the homogeneous and particular parts of the general solution.
1c. Solution of $4y'' + by' + 3y = 10\sin(4t)$

vary $b = 4, 5$ and $6$.

EDU>> solb4 = dsolve('4*D2y+4*Dy+3*y = 10*sin(4*t)','y(0) = -1','Dy(0)=1','t')

$\frac{305}{3977}\sin(\frac{1}{2}\sqrt{2}\cdot t)\cos(\frac{1}{2}(2\cdot(1/2)+8)\cdot t) - \frac{80}{3977}\cos(\frac{1}{2}\sqrt{2}\cdot t)\cos(\frac{1}{2}(2\cdot(1/2)-8)\cdot t) - \frac{305}{3977}\cos(\frac{1}{2}\sqrt{2}\cdot t)\sin(\frac{1}{2}(2\cdot(1/2)+8)\cdot t) - \frac{80}{3977}\sin(\frac{1}{2}\sqrt{2}\cdot t)\cos(\frac{1}{2}(2\cdot(1/2)-8)\cdot t) - \frac{335}{7954}\sqrt{2}\cdot \cos(\frac{1}{2}\sqrt{2}\cdot t)\cos(\frac{1}{2}(2\cdot(1/2)-8)\cdot t) - \frac{1260}{3977}\sqrt{2}\cdot \sin(\frac{1}{2}\sqrt{2}\cdot t)\sin(\frac{1}{2}(2\cdot(1/2)-8)\cdot t)$

EDU>> ezplot(solb4, [0 10])

EDU>> hold on

EDU>> solb5 = dsolve('4*D2y +5*Dy + 3*y= 10*sin(4*t)','y(0)=-1','Dy(0)=1');
solb5 =

$-\frac{610}{3977}\sin(4\cdot t) - \frac{160}{3977}\cos(4\cdot t) - \frac{3817}{3977}\exp(-\frac{1}{2}\cdot t)\cos(\frac{1}{2}(2\cdot(1/2)-8)\cdot t) + \frac{9017}{7954}\exp(-\frac{5}{8}\cdot t)\sin(\frac{1}{8}\cdot 23\cdot (1/2)\cdot t)$

EDU>> ezplot(solb5, [0 10])
EDU>> solb6 = dsolve('4*D2y +6*Dy + 3*y = 10*sin(4*t)','y(0)=-1','Dy(0)=1'); solb6 =

simple(solb6)

solb6 =

-610/4297*sin(4*t)-240/4297*cos(4*t)-4057/4297*exp(-3/4*t)*cos(1/4*3^(1/2)*t)+14777/12891*exp(-3/4*t)*sin(1/4*3^(1/2)*(1/2)*t)*3^(1/2)

EDU>> ezplot(solb6, [0 10])
1d. Solution of $4y'' + 4y' + ky = 10\sin(4t)$

vary $k = 1, 2$ and $3$.

EDU>> solk1 = dsolve('4*D2y +4*Dy + 1*y= 10*\sin(4*t)','y(0)=-1','Dy(0)=1'); solk1 =
solk1 =

\[-\frac{32}{845}\cos(4t) - \frac{126}{845}\sin(4t) - \frac{813}{845}\exp(-\frac{1}{2}t) + \frac{29}{26}\exp(-\frac{1}{2}t)t\]

EDU>> ezplot(solk1, [0 10])
EDU>> hold on

EDU>> solk2 = dsolve('4*D2y +4*Dy + 2*y= 10*\sin(4*t)','y(0)=-1','Dy(0)=1'); solk2 =
solk2 =

\[-\frac{8}{205}\cos(4t) - \frac{31}{205}\sin(4t) - \frac{197}{205}\exp(-\frac{1}{2}t) \cos(\frac{1}{2}t) + \frac{461}{205}\exp(-\frac{1}{2}t) \sin(\frac{1}{2}t)\]

EDU>> ezplot(solk2, [0 10])

EDU>> solk3 = dsolve('4*D2y +4*Dy + 3*y= 10*\sin(4*t)','y(0)=-1','Dy(0)=1'); solk3 =
solk3 =

\[-\frac{610}{3977}\sin(4t) - \frac{160}{3977}\cos(4t) - \frac{3817}{3977}\exp(-\frac{1}{2}t) \cos(\frac{1}{2}2^{1/2}t) + \frac{9017}{7954}\exp(-\frac{1}{2}t) \sin(\frac{1}{2}2^{1/2}t)2^{1/2}\]

EDU>> ezplot(solk3, [0 10])