Extra Credit 1 in MA 341.002

Due Date is Thursday, September 9.
How much extra credit? Up to 10 percent on exam one.

Consider the initial value problem

$$y'(t) = .01(y - 1)(y - 10) \text{ and } y(0) = 8.$$

Consult the Matlab/Maple links for lectures 3, 4 and 8 in the old syllabus:

http://www4/eos/users/w/white/www/white/ma341/ma341hp.htm

(a). By hand calculations and the separations of variables method find the exact algebraic solution.

(b). Use *dsolve* and *ezplot* to find this solution and its graph.

(c). Use *quiver* to find the direction field and use it to approximate the graphical solution.

(d). Use the Euler numerical method to find an approximate numerical solution. Yo may wish to modify the code in lecture 8, *eulerc1.m.*

1a

$$\frac{dy}{dt} = c(y-1)(y-10), \quad y(0) = 8, \quad c = .01$$

$$\frac{dy}{(y-1)(y-10)} = c\,dt \qquad 1 < y < 10 \text{ for "small" } t$$

$$\int \frac{dy}{(y-1)(y-10)} = \int c\,dt + C_1$$

$$\int -\frac{1}{9}\frac{1}{y-1} + \frac{1}{9}\frac{1}{y-10}\,dy = ct + C_1$$

$$-\frac{1}{9}\ln|y-1| + \frac{1}{9}\ln|y-10| =$$

$$-\frac{1}{9}\ln(y-1) + \frac{1}{9}\ln(10-y) =$$

$$\ln\left(\frac{10-y}{y-1}\right) = 9ct + 9C_1$$

$$e^{\ln\left(\frac{10-y}{y-1}\right)} = e^{9ct+9C_1}$$

$$\frac{10-y}{y-1} = e^{9ct}\underbrace{e^{9C_1}}_{\hat{C}}$$

$$\frac{10-y}{y-1} = \hat{C}e^{9ct}$$

$$10-y = (y-1)\hat{C}e^{9ct}$$

$$= y\hat{C}e^{+9ct} - \hat{C}e^{+9ct}$$

$$y(-1 - \hat{C}e^{9ct}) = -10 - \hat{C}e^{9ct}$$

$$y = \frac{10 + \hat{C}e^{9ct}}{1 + \hat{C}e^{9ct}}$$

$$y(0) = 8 = \frac{10 + \hat{C}\cdot 1}{1 + \hat{C}}$$

$$8(1+\hat{C}) = 10 + \hat{C}$$

$$7\hat{C} = 2$$

$$\hat{C} = 2/7$$

$$y = \frac{10 + \frac{2}{7}e^{9ct}}{1 + \frac{2}{7}e^{9ct}}$$

$$\frac{1}{(y-1)(y-10)} = \frac{A}{y-1} + \frac{B}{y-10}$$

$$1 = A(y-10) + B(y-1)$$

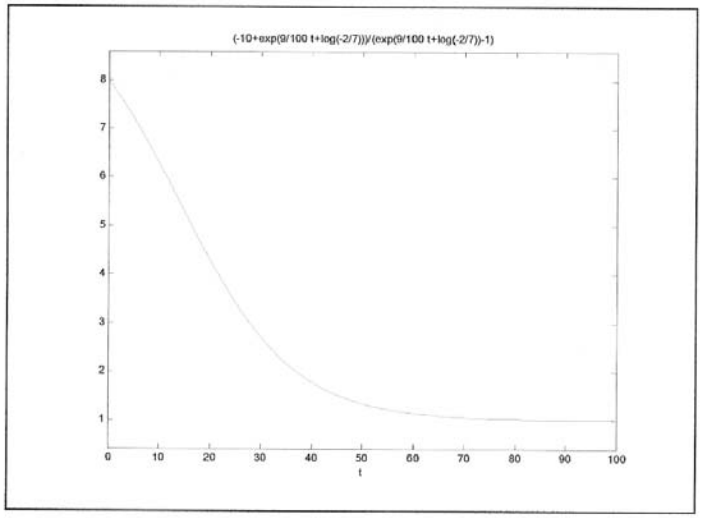
$y \to 10 \Rightarrow \quad 1 = 0 + B\cdot 9$

$B = 1/9$

$y = 1 \Rightarrow \quad 1 = A(-9) + 0$

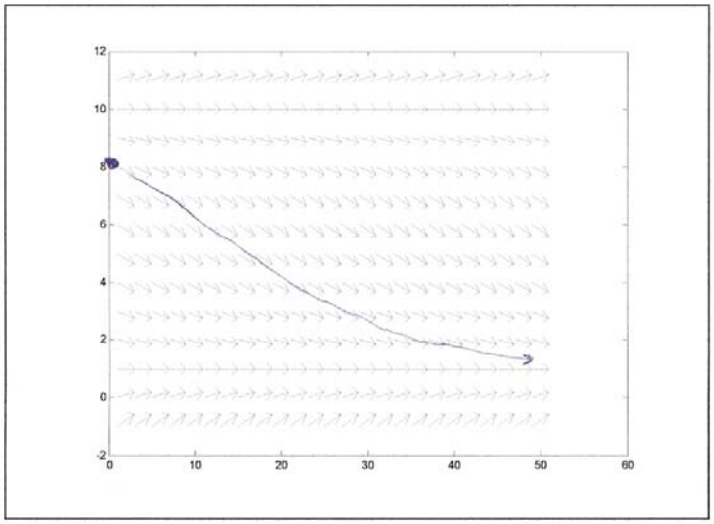
$A = -\frac{1}{9}$

```
EDU>> sol=dsolve('Dy=.01*(y-1)*(y-10)','y(0)=8','t')

sol =

(-10+exp(9/100*t+log(-2/7)))/(exp(9/100*t+log(-2/7))-1)

EDU>> pretty(sol)

                -10 + exp(9/100 t + log(-2/7))
                ------------------------------
                 exp(9/100 t + log(-2/7)) - 1
EDU>> simplify(sol)

ans =

2*(35+exp(9/100*t))/(2*exp(9/100*t)+7)

EDU>> ezplot(sol,[0 40])
EDU>> ezplot(sol,[0 100])
```



```
EDU>> clear
EDU>>  [t y]=meshgrid(1:2:50,-1:1:11);
EDU>> slope = .01.*(y-1).*(y-10);
EDU>> quiver(t,y,ones(size(t)),slope)
```

$y(0) = 8$



```
clear;
y(1) = 8.;
yexact(1) = 8;
T = 50;
KK = 100
h = T/KK;
t(1)= 0.;
for k = 1:KK
   t(k+1) = t(k) + h;
   yexact(k+1) = 2*(35+exp(.09*t(k+1)))/(2*exp(.09*t(k+1))+7);
   y(k+1) = y(k) + h*.01*(y(k)-1)*(y(k)-10);
end
error = abs(yexact(KK+1) - y(KK+1))
plot(t,y,'b',t,yexact,'r')
```