

Lesson 8: Spread of Information and ode45

8.1 Applied Problem.

A marketing firm wants to know how fast a new product will be learned about. Information can be obtained either via mass media or via personal communication. The mass communication refers to newspapers, journals, radio and television. The personal communication refers to discussions with other people. The personal communications can have large changes in the number of informed people once a significant proportion of the population knows about the new product. In effect, every informed person can become a "broadcaster" of the information.

The marketing firm would like to know how to best inform a certain segment of the population about a given piece of information. On any day what number of people will be informed about the new product?

8.2 Differential Equation Model.

We may model the two modes of information dissemination. Moreover, we may consider a discrete model for small populations, or a continuum model for much larger populations. The four models for the spread of information are

- spread of information via mass communication and discrete,
- spread of information via mass communication and continuous,
- spread of information via personal communication and discrete and
- spread of information via personal communication and continuous.

Spread of Information via Mass Communication and Discrete.

This model is very similar to the cooling model. Let $y(i)$ be the total number who are informed by day i . Let $dt = 1$ day. And suppose the total population is 20000, and so, the uninformed group is equal to $20000 - y(i)$. If either dt or $20000 - y(i)$ are large, then we expect the change in the number who are informed to be large. One way to model this is to claim the change in the number who are informed is directly proportional to the product of the change in time, dt , and the number who do not know, $20000 - y(i)$,

$$y(i+1) - y(i) = dt * k * (20000 - y(i))$$

where k reflects the effectiveness of the mass media. This is the Euler model of the following continuous model.

Spread of Information via Mass Communication and Continuous.

The continuous model can be used for very large populations where $y(t)$ is viewed as a continuous function of time, t , and $y(t)$ is the percentage (or number) of people who know at any time t . In the previous model we considered the discrete model. Here we have a similar formula for the change in the number who know

$$y(t + dt) - y(t) \approx dt * k * (20000 - y(t)).$$

Now the number who know, $y(t)$, will change a little as t moves from t to $t+dt$, and this is the reason we do not have an equality sign for the above change in the number who know. If we let dt go to zero, we just get zero on both sides! However, if we divide by dt and let dt go to zero, we get the differential equation

$$y' = k (20000 - y).$$

This differential equation with known k and given initial condition $y(0) = y_0$ is the continuous model.

Spread of Information via Personal Communication and Discrete.

If information is being learned from those who already know, then the constant, k , in the above two models must be changed to reflect the number of the informed people. One reasonable choice might be $k = cy$ where c is another constant. Then the change in the number of people who know would be directly proportional to the product of the change in time, dt , the number who know, y , and the number who do not know

$$y(i+1) - y(i) = dt * c * y(i) * (20000 - y(i)).$$

This is the Euler model of the following continuous model.

Spread of Information via Personal Communication and Continuous.

A similar modification for the continuous model gives

$$y(t + dt) - y(t) \approx dt * c * y(t) * (20000 - y(t)).$$

Now divide by dt and let it go to zero to get the differential equation

$$y' = cy (20000 - y).$$

The differential equation with c and an initial condition $y(0) = y_0$ make up the continuous model. A differential equation of the form

$$y' = cy(M - y)$$

is called a **logistic differential equation**, and it and its variations are an important class of models.

8.3 Method of Solution.

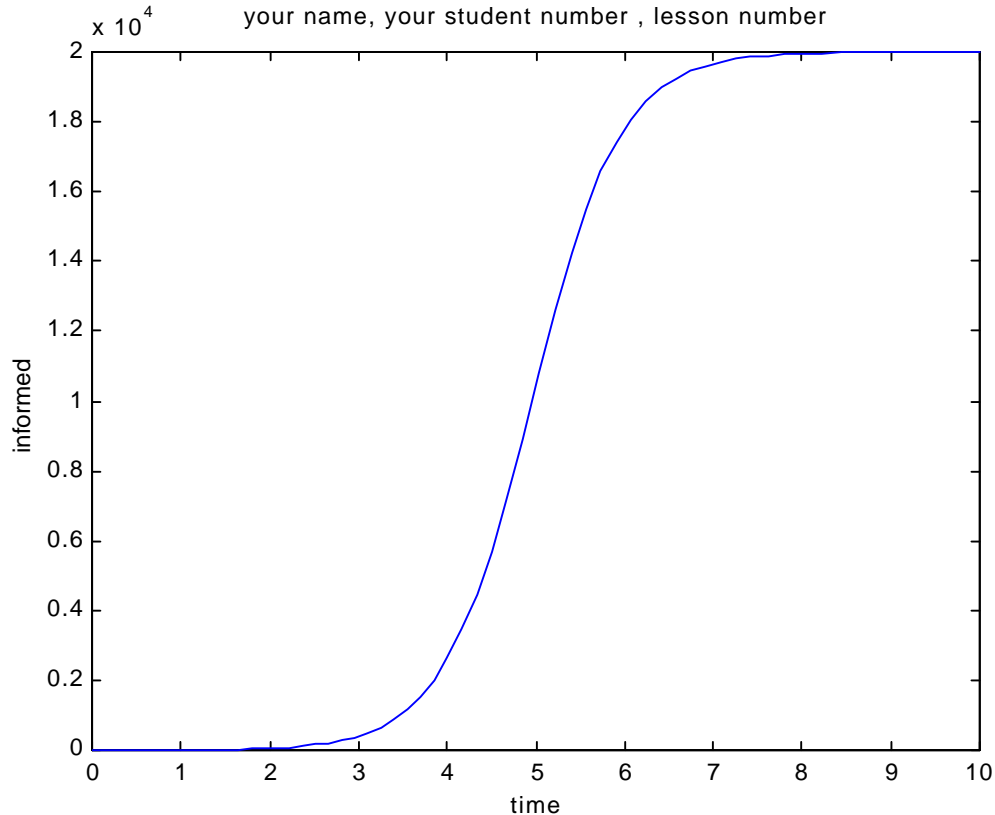
In this lesson we will use the Matlab command `ode45` to solve our differential equation. This command is a robust implementation of a variable step size method and the fourth and fifth order Runge-Kutta method. Matlab has a number of solvers for initial value problems, and the reader can learn about these by either executing the Matlab command `odedemo` or using `helpdesk` and looking under differential equations.

8.4 Matlab Implementation.

An easy way to execute the `ode45` command is to create two m-files: **`ypinfo.m`** and **`info.m`**. The `ypinfo.m` file contains the right side of the differential equation for the personal communication model. In the second line of the `ypinfo.m` file we have the logistic model with harvesting where $c = .0001$ and $M = 20000$; here the symbol $y(1)$ represents the number who are informed out of population equal to 20000. The c was computed as given in the next section from two observations: initially one person knew and after one day 200 people knew. The output is given in the graph below.

```
function ypinfo = ypinfo(t,y)
    ypinfo(1) = (.0001)*y(1)*(20000 -y(1));

%your name, your student number, lesson number
clear;
yo = [1];
to = 0;
tf = 10;
[t y] = ode45('ypinfo',[to tf],yo);
plot(t,y)
title('your name, your student number, lesson number')
xlabel( 'time')
ylabel( 'informed')
```



8.5 Numerical Experiments.

Suppose $y(0) = 1$ and $y(1) = 200$ are the number who know initially and after one day. Then for the mass communication model we can find k by using $y' = k(20000 - y)$ and the improved Euler method

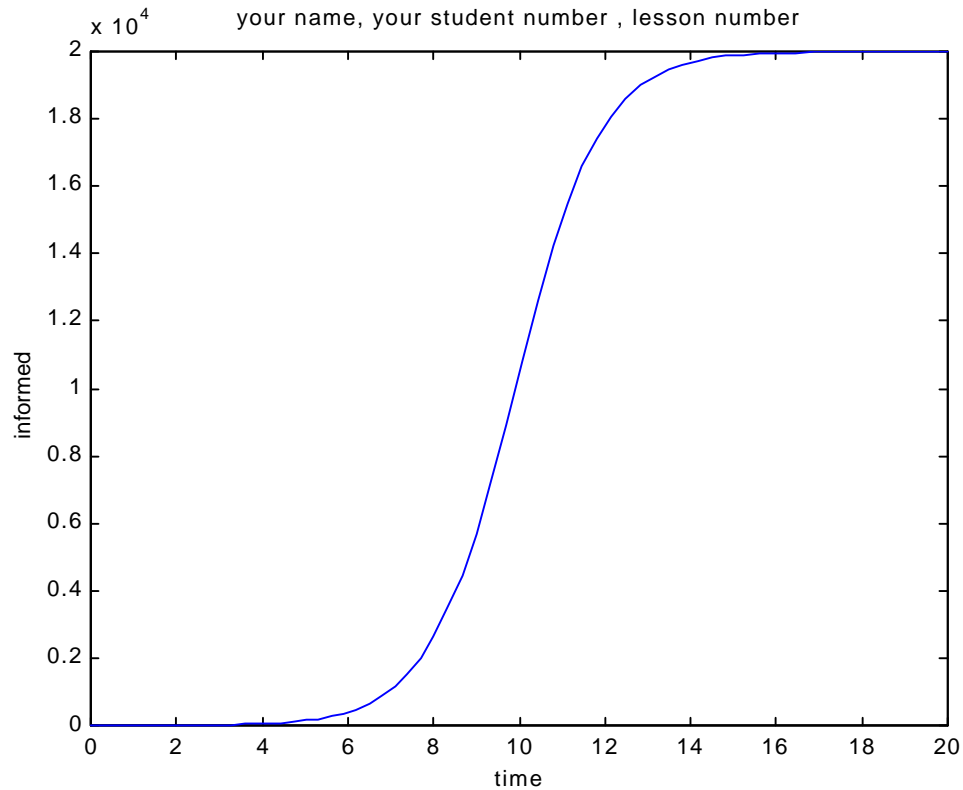
$$200 - 1 = 1 * k * (1/2) * ((20000 - 1) + (20000 - 200)) \text{ and } k = .01.$$

For the personal communication model we find the c by using the logistic differential equation $y' = cy(20000 - y)$ and the improved Euler method

$$200 - 1 = 1 * c * (1/2) * (1 * (20000 - 1) + 200 * (20000 - 200)) \text{ and } c = .0001.$$

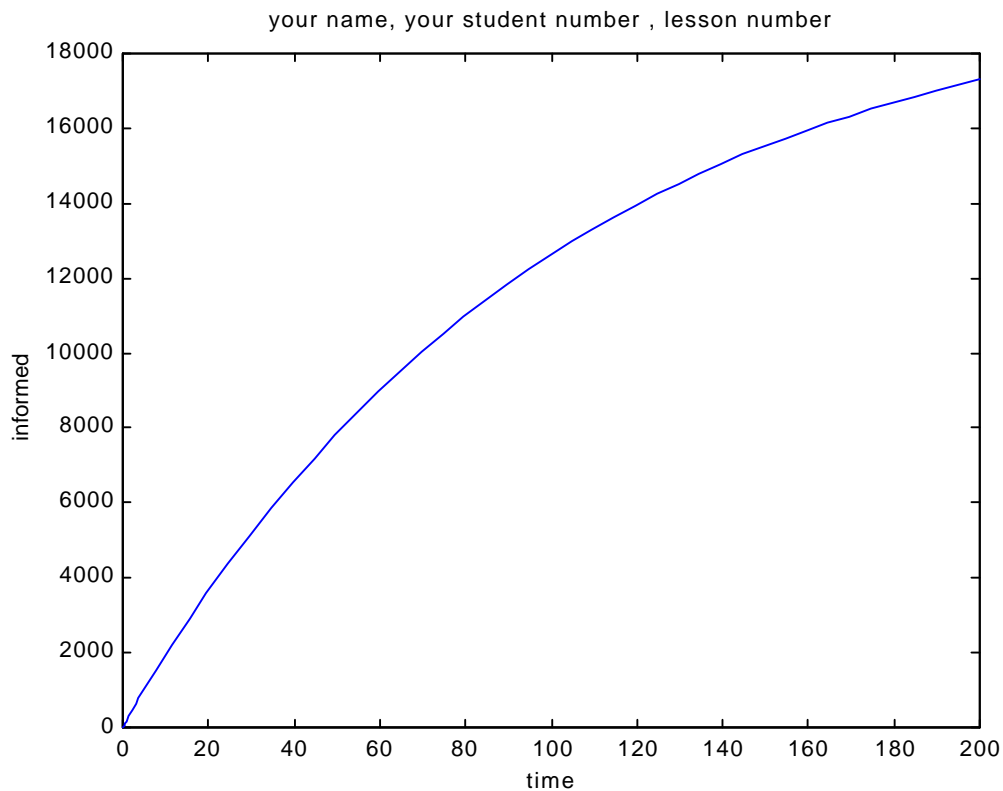
In the above graph, for the personal communication model, most people were informed after seven days. If the constant c , the "gossip" constant, is smaller, then it would take longer for the entire populations to learn the information. The second graph

is for $c = .00005$ and not $.0001.$, and the final time is changed from 10 to 20. Here it takes about 14 days for most of the population to become informed.



The next calculation is for the mass media communication model where $y' = .01(20000 - y)$. Note it takes much longer for most of the population to learn the information. In the graph below not even 90% were informed after 200 days. The $k = .01$ is the "advertising" constant, and if it is increased, then most of the population will learn the information sooner.

Many advertisements are very clever or cute. This is to encourage people to "talk" about the advertisement so that **both** the mass media and personal communication modes of learning are used.



8.6 Additional Calculations.

Consider the personal communication model where the population is measured in percent so that $M = 100$. Assume initially 1 percent know the information so that $y(0) = 1$. Assume after one day $1+S$ percent know the information.

- (a). Find the c in the logistic equation by using the improved Euler method.
- (b). Modify the `ypinfo.m` file.
- (c). Insert your name and student number into the `info.m` file.

Execute the `info.m` file.

- (d). Print the `ypinf.m` and `info.m` files and the graphs.