

Differential Equation Model.

Continuous Version of Newton's Law of Cooling.

$$y' = c(ysur - y) \text{ and } y(0) = y_0.$$

Discrete Version of Newton's Law of Cooling.

$$y(i+1) - y(i) = c*(ysur - y(i))*dt.$$

We can solve for $y(i+1)$

$$\begin{aligned} y(i+1) &= y(i) + c*(ysur - y(i))*dt \\ &= (1 - c*dt)*y(i) + c*ysur*dt. \end{aligned}$$

Method of Solution.

Find c by using two observations

$$y(0) = 200 \text{ and } y(1) = 195.$$

Now apply the model with $i = 0$ to get

$$y(0+1) = (1 - c*dt)*y(0) + c*ysur*dt$$

$$195 = (1 - c *1)*200 + c*70*1.$$

So, solve for c to get

$$c = (195 - 200)/(70 - 200) = 5/130 = 1/26.$$

Experiment with different $n = KK = 4, 8, 16, 32$.

Compare the errors in the exact and numerical solutions

Exact solution at $t = 50$ is $y(50) = 89.00035$

Numerical solution at $t = 50$ is $Y(n+1)$

Error = $89.0035 - Y(n+1)$

$$= 9.55137 \quad \text{for } n = 4$$

$$= 4.58938 \quad \text{for } n = 8$$

$$= 2.24524 \quad \text{for } n = 16$$

$$= 1.11024 \quad \text{for } n = 32.$$

Matlab Implementation.

Use function file feul.m and m-file euler.c.m

```
function feul = feul(t,x)
    feul = .10*(70. - x);

%your name, your student number,
lesson number
clear;
y(1) = 200.;
T = 50;
KK = 100
h = T/KK;
t(1)= 0.;
for k = 1:KK
    y(k+1) = y(k) +
        h*feul(t(k),y(k));
    t(k+1) = t(k) + h;
end
plot(t,y)
title('your name, your student
number, lesson number')
xlabel('time')
ylabel('temperature')
```

your name, your student number, lesson number

