Differential Equation Model.

Left Loop Equation:

\[ L Q'' + R Q' + \frac{1}{C} Q = V(t) = M i' \]

and \( Q(0) \) and \( Q'(0) \) are given,

and \( i \) is current in the right loop.

Charge in the left loop = \( Q \).

Current in the left loop is \( I = Q' \).
Center Loop Equation:

\[(1/C) Q = x = \text{voltage drop in triode.}\]

Right Loop Equation:

\[i = a x - b x^3\] where the \(a\) and \(b\) are constants,

which are to be determined via curve fitting.

Note,

\[i' = a x' - b 3x^2x'\] and

\[i' = a \frac{I}{C} - b \frac{3(Q/C)^2}{I/C} I/C.\]
Upon scaling of time and the charge, we have

**Van der Pol’s DE:**

\[ y'' - \mu(1 - y^2)y' + y = 0. \]

### Method of Solution.

The equivalent coupled system is

\[ y_1' = y_2 \quad \text{with } y_1(0) = y(0) \text{ and} \]

\[ y_2' = \mu(1 - y_1^2)y_2 - y_1 \quad \text{with } y_2(0) = y'(0). \]

Matlab's ode23s and ode15s can be used to solve this system, which is stiff for larger \( \mu \).
Matlab Implementation.

```matlab
function ypvdpol = ypvdpol(t,y)
    ypvdpol(1) = y(2);
    ypvdpol(2) = 10*(1-y(1)^2)*y(2) - y(1);
    ypvdpol = [ypvdpol(1) ypvdpol(2)]';

% your name , your student number,
lesson number
clear;
tf = 100;  % choose tf ten times mu
yo = [2 0];
[t y] = ode23s(ypvdpol,[0 tf],yo);
plot(t,y(:,1));
title('your name, your student number,
      lesson number')
xlabel('time')
ylabel('charge')
%plot(y(:,1),y(:,2));
```
Numerical Experiments.

In the first calculation we use $\mu = 10$ and the final time should be about ten times $\mu$.

The graph is for the scaled charge versus time.

The graph for the scaled current has a series of spikes.

In the Van der Pol circuit this might be series of flashes if the resistance was a light bulb.

For larger $\mu$ this is a very stiff system.