Integrating Computation into Math/Science Education

By

Robert E. White
Department of Mathematics
NCSU
white@math.ncsu.edu
http://www4.ncsu.edu/~white

USE OF COMPUTATION TOOLS
IS
REQUIRED

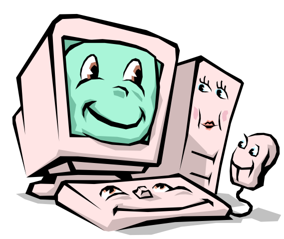
By Hand:

$$\int (ax+b)e^{2x}dx = a\int xe^{2x}dx + b\int e^{2x}dx$$
$$= a[xe^{2x}/2 - \int 1/2e^{2x}dx] + b\int e^{2x}/2(2dx)$$
$$= a[x/2e^{2x} - (1/2)^2 e^{2x}] + b/2e^{2x}$$
$$= e^{2x}(ax/2 - a/4 + b/2)$$

$Check:$

$$\frac{d}{dx}e^{2x}(ax/2 - a/4 + b/2) = e^{2x}2(ax/2 - a/4 + b/2)$$
$$+ e^{2x}(a/2 - 0 + 0)$$
$$= (ax+b)e^{2x}$$

By Symbolic Computation:



```
>> syms a b x
>> int((a*x+b)*exp(2*x),x)
      ans = 1/4*a*(2*exp(2*x)*x -  exp(2*x))
            + 1/2*exp(2*x)*b
```

Important Issues:

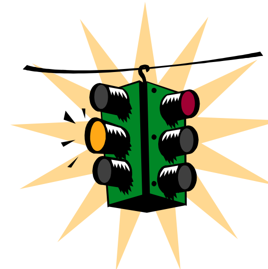
- Level of computer use

    1. Limited in class demonstrations

    2. In class demonstration with out of class computer homework

    3. In class demonstrations and student in class computation

- **Correctness of computations**



1. students may be naïve or too trusting

2. easy to look for the quick answer

3. confirmation or reasonableness of "solution"

simple case by hand
duplicate with computer
enhanced problem

4. compare with physical observations
(empirical)

5. mathematical analysis
(deductive)

- **Costs of computations**

1. hardware
low end is about $500

2. software
low end is about $40 for Redhat:
older software such as open office, octav,
variety of programming languages,……

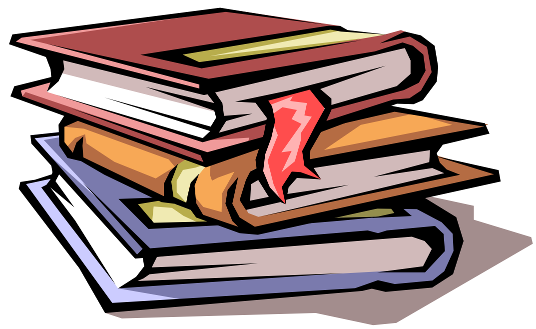
3. time sinks

student use and syntax errors
eats up traditional classroom time
faculty preparation and setup

Where does computation fit into the overall goal of education?

From party animal

To degree hostage



To life time learner

Six Computational Lessons

- **Application driven**
- **Includes traditional mathematics**
- **Computer implementation**
- **Vary parameters computations**
- **Two lessons use spreadsheets**
- **Two lessons use Matlab ode45 solver**
- **Two use Matlab programming language**

Credit Card Debt

Model:

$$u^{k+1} = u^k + \text{change in month } k$$
$$= u^k + r/12\, u^k - P + C$$

Math:

$$u^{k+1} = a\, u^k + b$$
$$= a\,(a\, u^{k-1} + b) + b$$
$$= a^2\, u^{k-1} + (a + 1)\, b$$
$$\vdots$$
$$= a^{k+1}\, u^0 + (a^k + \cdots + 1)\, b$$
$$= a^{k+1}\, u^0 + (1 - a^{k+1})/(1-a)\, b$$

So, if abs(a) < 1, then u^{k+1} converges to b/(1-a).
This generalizes to vector equations.

Computation:

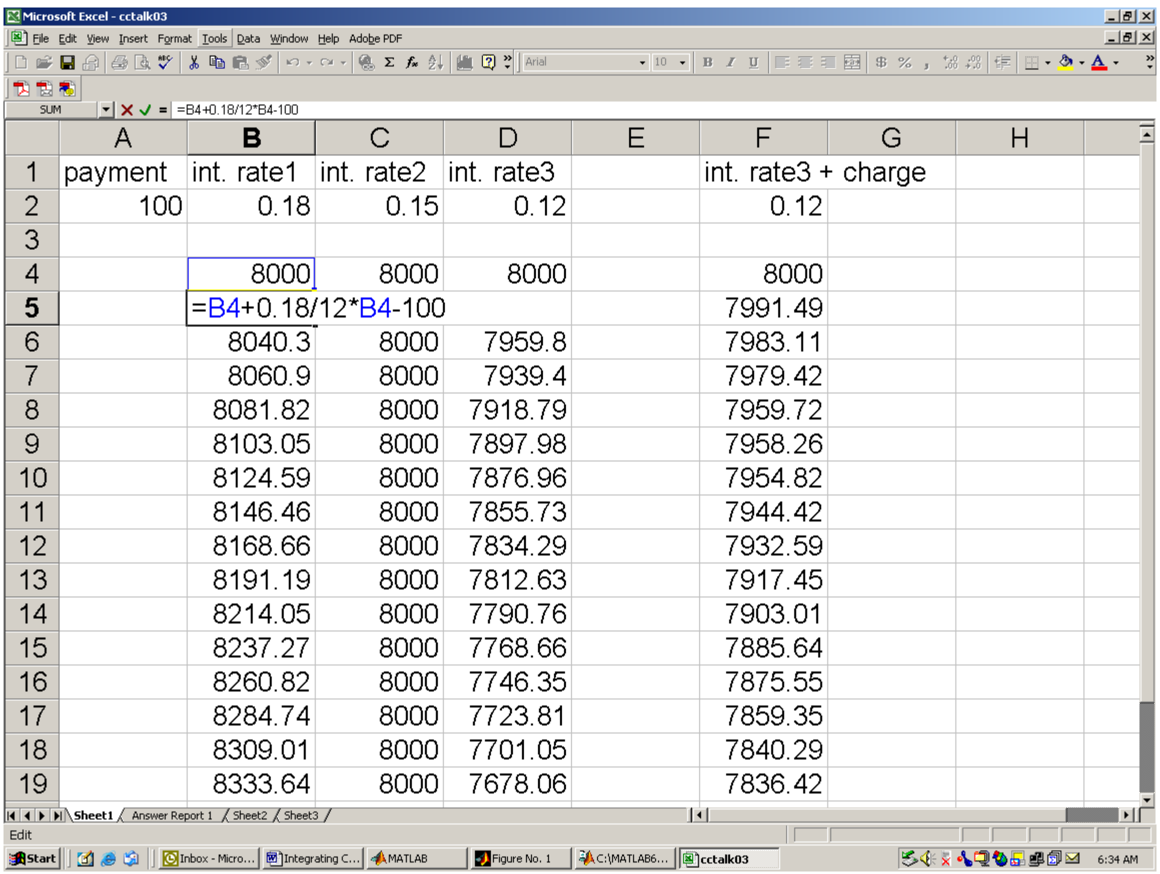
Could use spreadsheets or a programming language with loops.

Columns B,C and D are the standard plug and chug questions and column D uses

=D4+0.12/12*D4-100.

Column F uses random charges between 0 and 20

=F4+0.12/12*F4-100 + (20*RAND()).



	A	B	C	D	E	F	G	H
1	payment	int. rate1	int. rate2	int. rate3		int. rate3 + charge		
2	100	0.18	0.15	0.12		0.12		
3								
4		8000	8000	8000		8000		
5		=B4+0.18/12*B4-100				7991.49		
6		8040.3	8000	7959.8		7983.11		
7		8060.9	8000	7939.4		7979.42		
8		8081.82	8000	7918.79		7959.72		
9		8103.05	8000	7897.98		7958.26		
10		8124.59	8000	7876.96		7954.82		
11		8146.46	8000	7855.73		7944.42		
12		8168.66	8000	7834.29		7932.59		
13		8191.19	8000	7812.63		7917.45		
14		8214.05	8000	7790.76		7903.01		
15		8237.27	8000	7768.66		7885.64		
16		8260.82	8000	7746.35		7875.55		
17		8284.74	8000	7723.81		7859.35		
18		8309.01	8000	7701.05		7840.29		
19		8333.64	8000	7678.06		7836.42		

Maximum Profit in Two Markets

Let x = sales in market one and
y = sales in market two.

Profit = Revenue from market one
+ Revenue from market two
- Cost

$$= (97 - x/10)x + (83 - y/20)y - (20{,}000 + 3(x + y))$$

$$= 94x - .10\,x^2 + 80y - .05\,y^2 - 20{,}000.$$

Methods of Solution:

Complete the square

Graph the profit function

Use partial derivatives

Use solver in Excel

Formula in cell a3 is

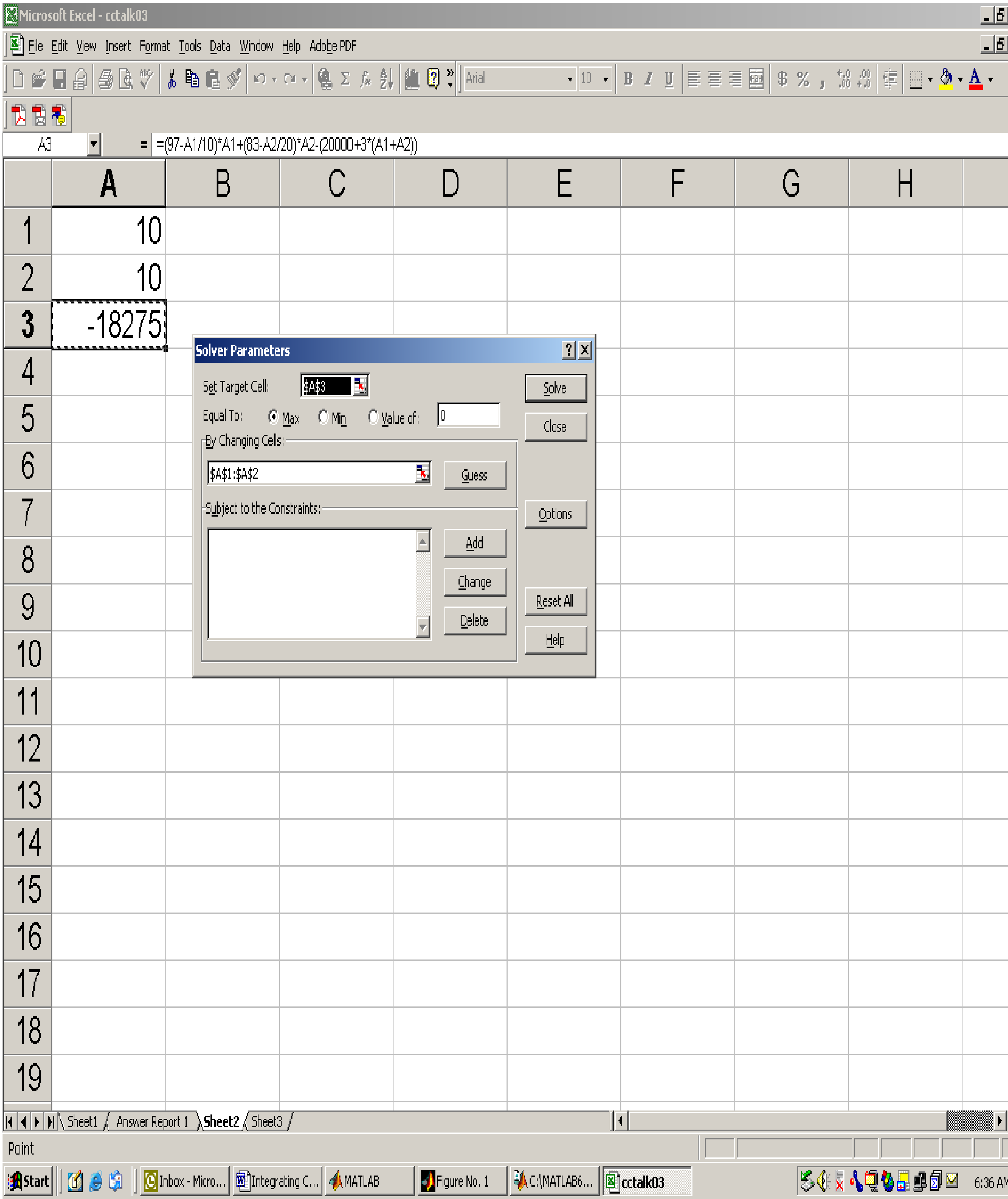
=(97-A1/10)*A1+(83-A2/20)*A2
-(20000+3*(A1+A2))

10
10
-18275

Solver is prompted to find the maximum and to vary cells a1:12

470.0001
799.9998
34090

One can impose inequality and equality constraints.


Microsoft Excel - cctalk03
File Edit View Insert Format Tools Data Window Help Adobe PDF
A3
=(97-A1/10)*A1+(83-A2/20)*A2-(20000+3*(A1+A2))
A B C D E F G H
1 10
2 10
3 -18275
Solver Parameters
Set Target Cell: $A$3
Equal To: Max Min Value of: 0
By Changing Cells:
$A$1:$A$2
Guess
Subject to the Constraints:
Add
Change
Delete
Solve
Close
Options
Reset All
Help
Sheet1 Answer Report 1 Sheet2 Sheet3
Point
Start
Inbox - Micro...
Integrating C...
MATLAB
Figure No. 1
C:\MATLAB6...
cctalk03

Population with Harvesting

y' = c(M-y)y – h(t)

Methods of Solution:

Direction fields

Separation of variables when h(t) is constant

Euler methods

Variable step Runge-Kutta 45

Use Matlab ode45

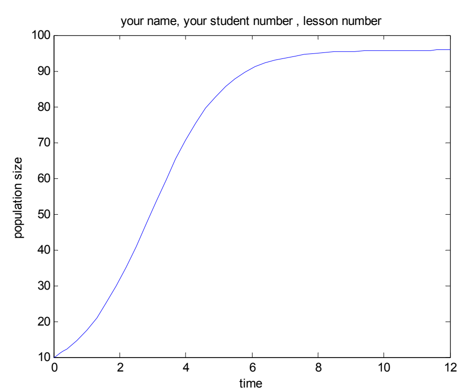
Matlab m-file ypfishh.m:

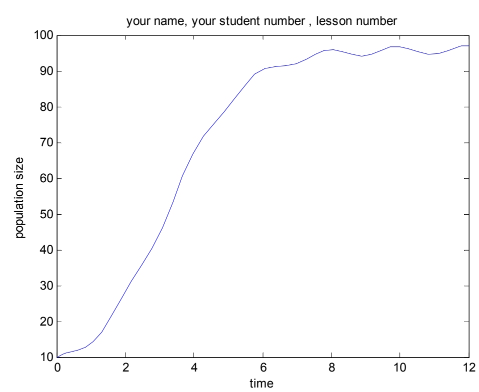
```
function ypfishh = ypfishh(t,y)
  ypfishh(1) = (.01)*y(1)*(100 -y(1))-4.0;

                 % 4.*(1+sin(pi*t)
```

Matlab m-file fishh.m:

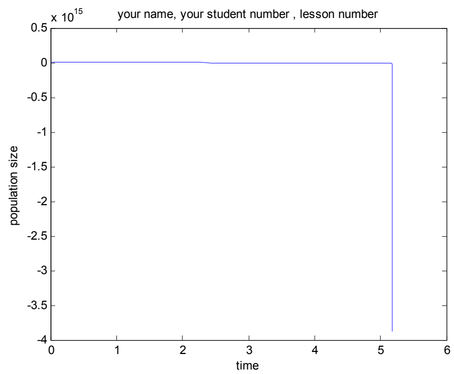
```
clear;
yo = [10];
to = 0;
tf = 12;
[t y] = ode45('ypfishh',[to tf],yo);
plot(t,y)
```



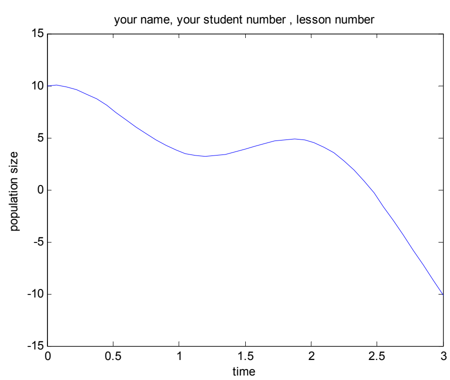


In the constant harvesting case there is the possibility of over harvesting. Perhaps, this is also true for the variable harvesting. Try

$$h(t) = 8*(1 + \sin(pi*t)).$$



Use a smaller final time, tf.



LRC Tuning

$LQ'' + RQ' + (1/C)Q = V(t)$

Special case is $V(t) = V_0 \cos(wt)$ whose solution is

$$I(t) = C_1 r_1 \exp(r_1 t) + C_2 r_2 \exp(r_2 t)$$

$$+ E (R^2 + (1/(C\omega) - (L\omega))^2)^{-1/2} \cos(\omega t + \phi).$$

The homogeneous solution decays to zero.

The particular solutions will be a maximum if

$E (R^2 + (1/(C\omega) - (L\omega))^2)^{1/2}$ is a minimum.

So, adjust C and L so that

$$1/(C\omega) - (L\omega) = 0.$$

Convert the second order ODE to a first order linear system and use ode45 to approximate the solution

y_1 = Q = charge and y_2 = Q' = I = current.

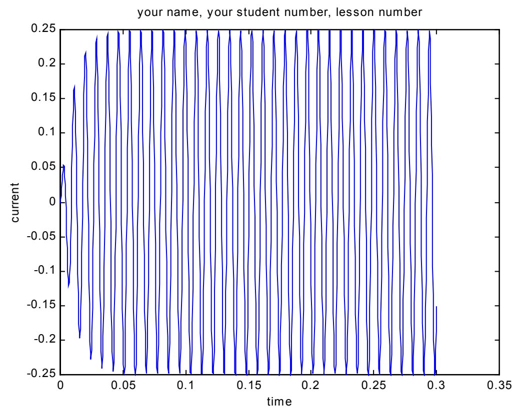
Matlab file yptune.m

```
function yptune = yptune(t,y)
 yptune(1) = y(2);
 yptune(2) = (100*cos(710.*t)-
            400*y(2)-10^6)*y(1))/2.0;
 yptune = [yptune(1) yptune(2)]';
```

Matlab file tune.m

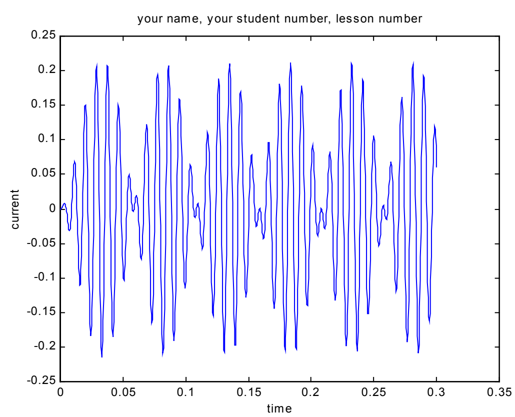
```
%your name, your student number, lesson number
clear;
t0 = 0;
tf = .3;
y0 = [0 0];
[t y] = ode45('yptune', [t0 tf],y0);
plot(t,y(:,2))
```

The output is consistent with the analytic solution



Now replace V(t) with

$$V(t) = 100 \sin(63t) \sin(710t).$$



Vary L and C to see the amplitude decrease (not tuned).

Pollutant in a Steam

Let $u_i^k \approx u(i\Delta x, k\Delta t)$ = concentration of pollutant.

Assume the vel = velocity of the stream is positive.

Discrete Model:

$$(\Delta x\ A)\ u_i^{k+1} = (\Delta x\ A)\ u_i^k + (\Delta t\ \text{vel}\ A)\ u_{i-1}^k$$

$$-\ (\Delta t\ \text{vel}\ A)\ u_i^k - (\Delta x\ A)\ \Delta t\ \text{dec}\ u_i^k$$

Matrix form is

$$u^{k+1} = \mathbb{A}u^k + b \in \mathbb{R}^n$$

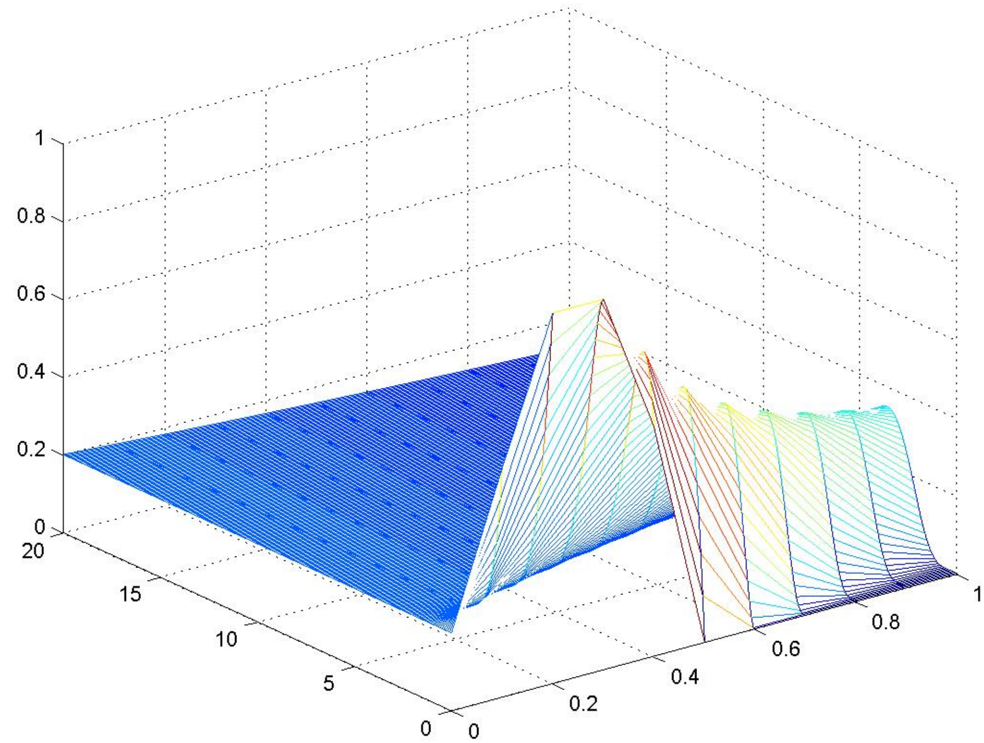
Continuous Model:

$u_t = -\text{vel}\ u_x - \text{dec}\ u$ with $u(x,0)$ and $u(0,t)$ given

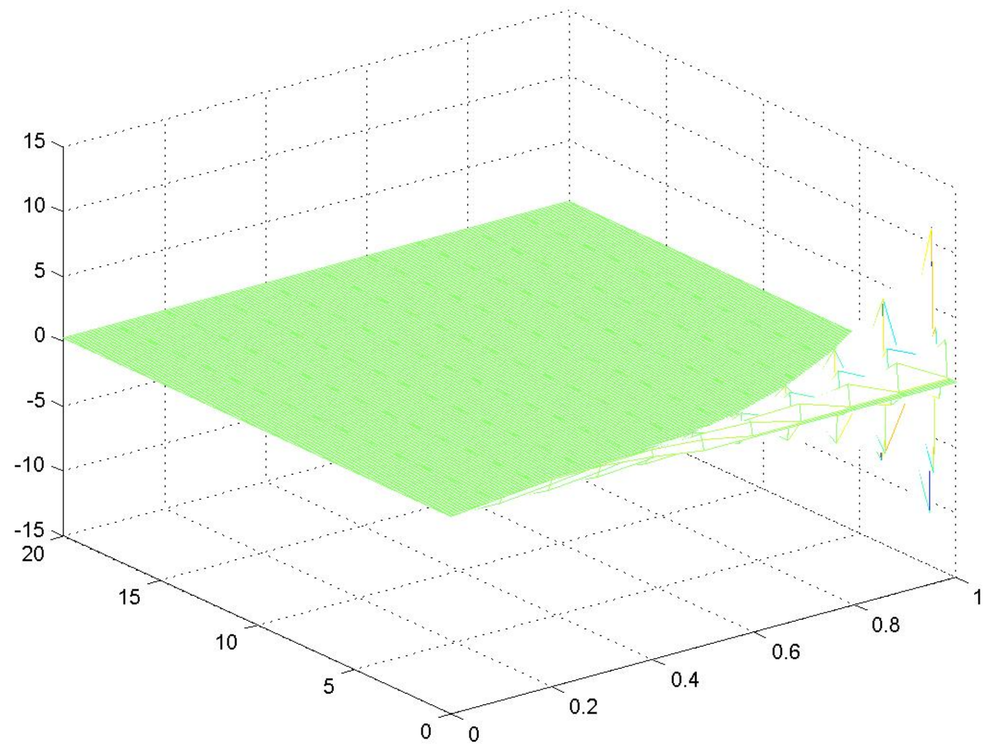
Matlab m-file flow1d.m

```
% This a model for the concentration of a pollutant.
% Assume the stream has constant velocity.
clear;
L = 1.0;    % length of the stream
T = 20.;    % duration of time
K = 200;    % number of time steps
dt = T/K;
n = 10.;    % number of space steps
dx = L/n;
vel =.2;   % velocity of the stream
decay = .1; % decay rate of the pollutant
for i = 1:n+1   % initial concentration
        x(i) =(i-1)*dx;
        u(i,1) =(i<=(n/2+1))*sin(pi*x(i)*2)+(i>(n/2+1))*0;
end
for k=1:K+1 % upstream concentration
        time(k) = (k-1)*dt;
        u(1,k) = -sin(pi*vel*0)+.2;
end
%
% Execute the finite difference algorithm.
%
for k=1:K   % time loop
        for i=2:n+1 % space loop
                u(i,k+1) =(1 - vel*dt/dx -decay*dt)*u(i,k) +
                                  vel*dt/dx*u(i-1,k);
        end
end
mesh(x,time,u')
% contour(x,time,u')
% plot(x,u(:,1),x,u(:,51),x,u(:,101),x,u(:,151))
```

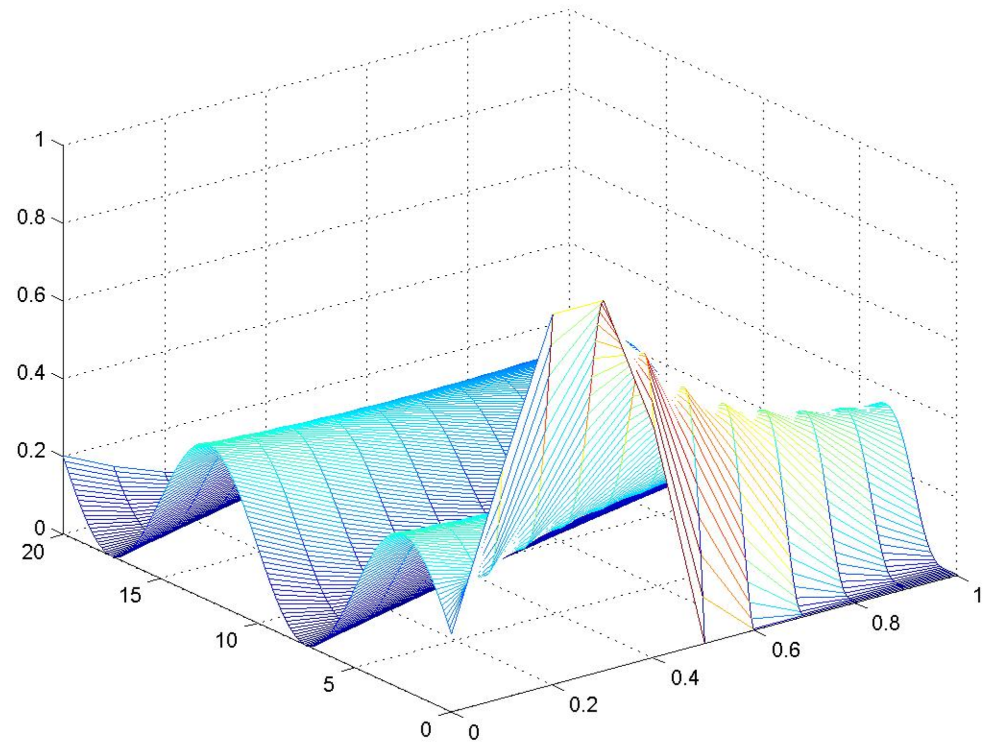
Vel = .2 and stable computation



Vel = 1.3 is unstable computation
(need smaller time step)



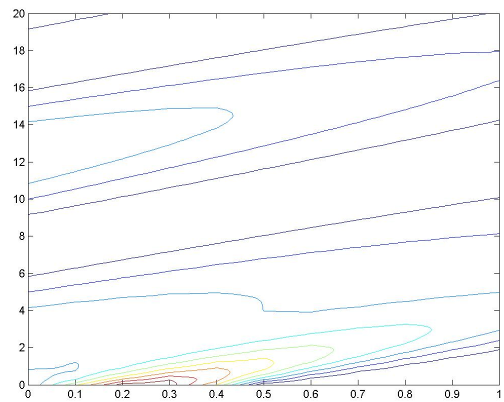
Vel = .2 and u(0,t) = (1+sin(πt))*.2



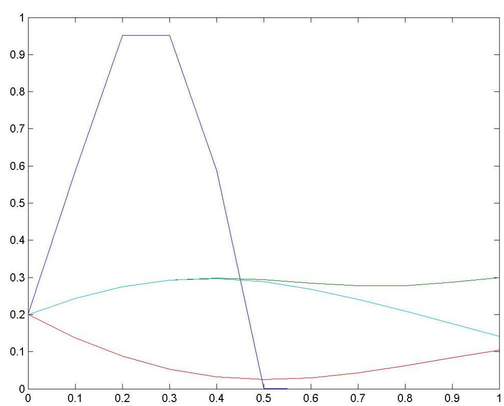
One needs to systematically vary the physical and numerical parameters of the model.

One should use alternative graphical output such as

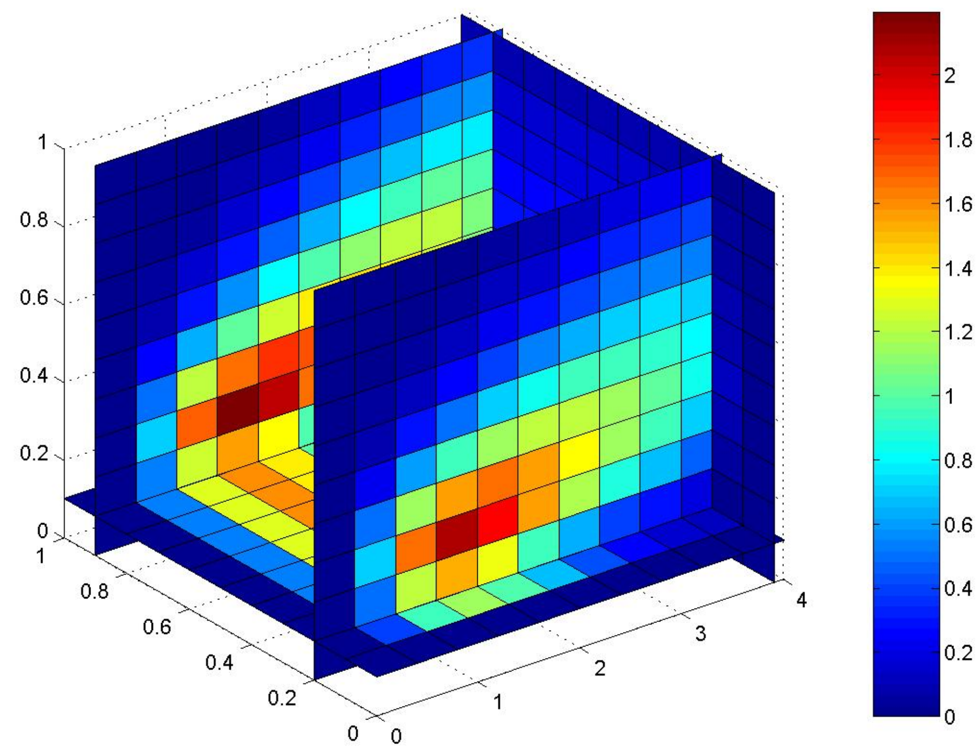
```
contour(x,time,u')
```




```
plot(x,u(:,1),x,u(:,51),x,u(:,101),x,u(:,151))
```



This can be extended to three dimensions where the Matlab command slice can be used to depict the concentration in space and time.



Concluding Remarks

- **In class demonstrations by the instructor in any math/science course can be done without too much effort.**

- **The demonstrations can be prepared in advance and put on transparencies.**

- **If computation is not a regular topic in the syllabus, then extra credit computation problems can be assigned.**

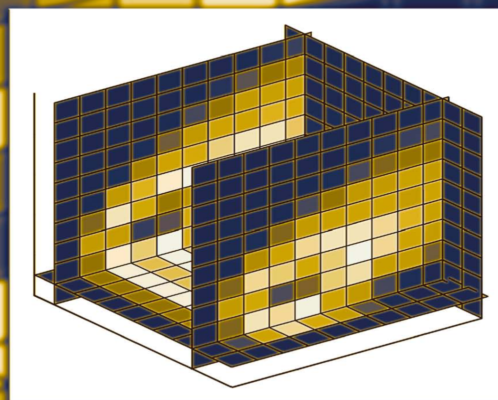
- **The models should be used to investigate the dependence on various parameters. This often generates sequences of computations and not just one or two computations.**

- **There should be assessments and extension of the models. One should attempt to develop learning based on curiosity and computational experimentation.**

- **Finally, a short advertisement ……**

COMPUTATIONAL MATHEMATICS

Models, Methods, and Analysis with MATLAB and MPI



ROBERT E. WHITE

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