MA 584
Suggested Projects
Due Thursday, December 9, 2004

Choose one project from the following:

1. Project of your own if you can find one by November 23.

2. Solve the Poisson equation

$$\nabla \cdot \beta \nabla u - qu = f(x, y) \tag{1}$$

on an irregular domain, for example, bounded by $x^2/a^2 + y^2/b^2 = 1$, or $r = 0.5 + 0.1 \sin(5\theta)$ in polar coordinates.

3. Implement, debug and test one of fast algorithms for the Poisson equations from the handout: *Fast Poisson Solvers by P. Swartztrauber*, for example, the FFT approach. You should assume a Neumann or mixed boundary condition at least along one side. Choose two examples with known exact solutions to test and debug your code. One of examples should be a quadratic functions and at least one of examples should have non-homogeneous boundary conditions.

4. Derive and implement the ADI method (alternating directional implicit) method for the heat equation with variable coefficients

$$u_t = (a(x, y, t)u_x)_x + (a(x, y, t)u_y)_y + f(x, y, t)$$

assuming that $a(x, y, t) > 0$, one example is $a(x, y, t) = 1 + \log(t^2 + x^2 + y^2)$. You should assume a Neumann or mixed boundary condition at least along one side. Choose two examples with known exact solutions to test and debug your code. At least one of examples should have non-homogeneous boundary conditions.

5. Implement a multi-grid method for 2D parabolic equations. Some notes and a sample code are available upon request.

6. Use a finite difference method to solve the non-linear minimal surface equation

$$\nabla \cdot \left(\frac{c(x, y)}{\sqrt{1 + |\nabla u|^2}} \nabla u \right) = f(x, y). \tag{2}$$

7. Use the Clawpack to solve a hyperbolic system:

`http://www.amath.washington.edu/~claw/`

You can use the examples there as well, for example, the traffic flow models. Analyze the numerical method, your computed results, and in connection with PDE theory if possible. (Reference: R. J. LeVeque, Numerical Methods for Conservation Laws, Birkhauser-Verlag, 1990).

Guidelines for the project, applicable to some of homework problems as well.

- A title of the project.
- Author(s) and affiliation.
- Abstract.
- Key words.
- Introduction (background information, the problem, motivation, and possible applications).
- Numerical method and theoretical analysis (consistency, stability, order of convergence, the computational cost and storage etc.)
- Numerical experiments and analysis using tables and plots: for example, the grid refinement analysis in table or plot form, selected error and solution plots, and any conclusions you can draw from your results. You need to label all you tables and plots.
- Conclusion and future work, acknowledgment.
- Reference.

Note that, it is OK to skip some of items if they do not apply to your problems. You can talk to the instructor for the references and other information.