1. Given four points in an O-XYZ coordinates:
   \[ A(0, 1, 0), \quad B(0, -1, 1), \quad C(1, 1, -1), \quad D(2, 0, -1). \]
   (a) Find the distance between B and D.
   (b) Find the vectors \( \vec{BA} \) and \( \vec{BC} \).
   (c) Find the dot product \( \vec{BA} \cdot \vec{BC} \) and the angle between the vectors \( \vec{BA} \) and \( \vec{BC} \).
   (d) Find the cross product \( \vec{BA} \times \vec{BC} \) and the area of the parallelogram formed by the vectors \( \vec{BA} \) and \( \vec{BC} \). Are A, B, and C in a straight line?
   (e) Find the equation of the sphere that centered at B with the radius 3.
   (f) Find the equation of line passing through B and C in parametric form, symmetric form, and the equation(s) (without parameter).
   (g) Are A, B, C, D in a same plane? (use triple product.)
   (h) Find the equation of the plane passing through A, B, and C. Find the distance between D and the plane.

2. Give a geometrical or physical meaning of the following integrals:
   \[
   \int_C ds, \quad \int_D dxdy, \quad \int_S ds, \quad \int_C \vec{F} \cdot dr, \quad \int\int_V dxdydz, \\
   \int\int_V \sigma dxdydz, \quad \int\int_S \vec{V} \cdot d\vec{S}.
   \]
   where \( C \) is a curve, \( S \) is a surface, \( D \) is a domain on \( xy \) plane, \( V \) is a solid in \( xyz \) space, \( \vec{F} \) is a
   force, \( \sigma \) is a density function, \( \vec{V} \) is a velocity function.

3. Given \( \mathbf{r}(t) = (\cos t, \sin t, t), \quad 0 \leq t \leq 4\pi. \)
   (a) Find the tangent direction at \( (0, 1, \pi/2) \). Find the equation of the tangent plane at this point.
   (b) Find the length of the curve between 0 and 2\( \pi \).

4. Give examples, equations, of the following surfaces. Ellipsoid, elliptic paraboloid, hyperbolic hyperboloid, cones, spheres, half planes.

5. Given \( f(x, y) = \frac{xy}{x^2 + y^2}, \) does \( f(x, y) \) has limit at \((-1, 1)\) at \((0, 0)\)? Is \( f(x, y) \) continuous?

6. Given
   \[ f(x, y) = \sqrt{9 - x^2 - y^2} \]
(a) Find the domain and range of \( f(x, y) \). Is \( f(x, y) \) continuous on the domain? Explain the definition of the continuity.

(b) Sketch the level curves of \( f(x, y) \), i.e. \( f(x, y) = k \), for \( k = 0, 1, 2, 3 \).

(c) Sketch the graph of \( f(x, y) \).

(d) Find \( \frac{\partial f}{\partial x} \), \( \frac{\partial f}{\partial y} \), \( \frac{\partial^2 f}{\partial x^2} \), \( \frac{\partial^2 f}{\partial y^2} \), and \( \frac{\partial^2 f}{\partial x \partial y} \). Under what kind of condition(s), we can conclude that \( \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \)?

(e) Find the equation of the tangent plane of the graph \( z = f(x, y) \) at \( x = 1 \) and \( y = 1 \).

(f) Given \( \mathbf{a} = -\mathbf{i} + 2 \mathbf{j} \), find the directional derivative of \( f(x, y) \) along the direction. Does \( f(x, y) \) increase or decrease along this direction?

(g) In what direction does \( f \) has the maximum rate of increase and decline?

7. If \( z = x/y, \ x = re^{st}, \ y = rse^{st} \). Evaluate the following:

\[
\frac{\partial z}{\partial r}, \ \frac{\partial z}{\partial s}, \ \frac{\partial z}{\partial t}, \ \frac{\partial z^2}{\partial r^2}.
\]

8. (a) Find all critical points of the function

\[ f(x, y) = 2x^4 - x^2 + 3y^2 + 4, \]

and then use the second derivative test to determine if each critical point corresponds to a local extreme value or saddle point of the function.

(b) Find the absolute maximum and minimum in the domain bounded by \( x = 0, \ y = 0, \) and \( x + 2y = 2 \).

(c) Find the equation of the tangent plane of the graph at \( (0,0,4) \).

9. (a) Find \( y' \) if \( x^3 + y^3 = 6xy \).

(b) Find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) if \( x^3 + y^3 + z^3 + 6xyz = 1 \).

10. Evaluate the following integrals.

(a) \[ \int_D f(x, y) \, dx \, dy, \text{ where } f(x, y) = xy, \ D = \{ (x, y), \ 0 \leq x \leq 2, \ 0 \leq y \leq 4 \}. \]

(b) \[ \int_0^1 \int_0^y 2x \, dx \, dy \, dz. \]

(c) \[ \int_V f(x, y, z) \, dx \, dy \, dz, \text{ where } V = \{ 0 \leq y \leq 1, \ 0 \leq z \leq y, \ 0 \leq x \leq y - z \}. \] Write it as an iterated integral of six different forms.

11. Given

\[ \int_0^2 dx \int_{\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (x^2 + y^2) \, dy. \]
(a) Change the order of integration, that is, integrate with $x$ first.
(b) Express and evaluate the integral using the poplar coordinates. Which approach is the easiest for this problem?

12. Express the double integrals $\int\int_D f(x, y)$ as two different iterated integrals both in Cartesian coordinates and in polar coordinates. Using the one which you think is simpler to evaluate the integrals. $f(x, y) = x \cos y$, $D$ is bounded by $y = 0$, $y = 0$, and $x^2 + 2x + y^2 = 0$.

13. Problem of Sample Quiz 1-4, Quiz 1-4.

14. If the density of the laminar $x^2 + 2x + y^2 \leq 0$ is $\rho(x, y) = \sqrt{x^2 + y^2}$, find the mass, the center*, and the inertia* about the origin, of the laminar.

15. Find the triple integral $\iiint_V x \, dV$, where $V$ is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$. Find the volume of $V$.

16. Express a triple integral as six different iterated integrals as in the Sample Quiz 4 and in Quiz 4.

17. Write down the spherical coordinates transformation and use it to evaluate the triple integral

$$\iiint_V xyz(x^2 + y^2 + z^2) \, dxdydz,$$

where $V$ is the solid between two spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 4$, that lies in the half space first octant, i.e., $x \geq 0$, $y \geq 0$, and $z \geq 0$.

18. Let $f(x, y, z) = x \sin y \, e^z$, $\vec{F} = \langle \sin x, \cos x, z^2 \rangle$.

(a) Find the gradient of $f(x, y, z)$, $\nabla f$. What is $\nabla \times \nabla f$?
(b) Find the divergence of $\vec{F}$, $\text{div} \vec{F}$. Is $\vec{F}$ divergence free?
(c) Find the curl of $\vec{F}$, $\text{curl} \vec{F}$. Is $\vec{F}$ irrotational? Is $\vec{F}$ conservative?
(d) Evaluate the line integral $\int_C \nabla f \cdot d\vec{r}$, where $C$ is the line segment from $(0, 1, 0)$ to $(1, \frac{\pi}{2}, 0)$.

19. Find the line integral $\int_C yds$, $\int_C yds$, $\int_C xds$, and $\int_C xdx$, where $C$ is the circle $x^2 + y^2 = 1$ in clockwise direction from $(0, 1)$ to $(1, 0)$ and the line segment from $(1, 0)$ to $(2, 3)$, and $-C$ is the same curve but in opposite direction.

20. (a) Use the Green’s theorem to evaluate the line integral

$$\frac{1}{2} \oint_C (-ydx + xdy)$$

where $c$ is the unit circle along the positive direction. What is the geometric meaning of your result?
(b) \[ \int_c (yx^2dx + x^2ydy) \]

21. Given \( \vec{F} = (y^2z^3, 2xyz^3, 3xy^2z^2) \).
   (a) Is \( \vec{F} \) conservative? Why?
   (b) If your answer is yes, find a potential function \( f(x, y, z) \) such that \( \nabla f = \vec{F} \).
   (c) Evaluate the line integral \( \int_c \vec{F} \cdot d\vec{r} \), where \( c \) is the helix \( (4\cos t, 2\sin t, t) \) from \( t = 4\pi \) to \( t = \pi/2 \).

22. (a) Give a parametric form of the surface \( x^2 + y^2 + z^2 = 4, \ y \geq 0 \).

   determine the range of the parameters.
   (b) Find the unit normal direction of the surface and the differential area.
   (c) Find the equation of the tangent plane at \( (0, 2, 0) \).
   (d) Find the area of the surface.
   (e) Find the integral \( \int_S f(x, y, z)ds \) and \( \int_S f(x, y, z)ds \).
   (f) Given a vector field \( \vec{F} = (2x, -yz, xy) \), find the flux of \( \vec{F} \), \( \int_S \vec{F} \cdot d\vec{s} \) and \( \int_S \vec{F} \cdot d\vec{s} \) across the surface. where \( -S \) is the same surface but with opposite normal direction.

23. Let \( \vec{F} = (3y^2z^3, 9x^6yz^2, -4xy^2) \).
   (a) Find the divergence of \( \vec{F} \).
   (b) Evaluate the flux of \( \vec{F} \) across the cube: \(-2 \leq x \leq 3, 0 \leq y \leq 1, -3 \leq z \leq -1 \).

24. Problems similar to Quizzes, class practices, homework, and examples explained in class.