COLLAPSING AND EXPLOSION WAVES IN PHASE TRANSITIONS WITH METASTABILITY, EXISTENCE, STABILITY AND RELATED RIEMANN PROBLEMS

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Abstract. Collapsing waves were observed numerically before and were used to explain the ring formations in dynamic flows involving phase transitions with metastability. In this paper, necessary and sufficient conditions for collapsing type of waves to exist are given. The conditions are that the wave speed of the collapsing wave is not less than a number and is supersonic on both sides of the wave. Existence and non-existence conditions for the explosion waves are also found. The stability of these waves are studied numerically. Although there are infinitely many collapsing (or explosion) waves for a fixed downstream state, the collapsing (or explosion) wave appeared in the solution of Riemann problem is numerically verified to be the one with the slowest speed. Although a Riemann problem in the zero visocity limit may have two solutions, one with, the other without, a collapsing (or explosion) wave, from the vanishing viscosity point of view, the one with a collapsing (or explosion) wave is numerically verified to be admissible.

1. Introduction

Dynamic flows involving liquid/vapour phase transition is an important phenomenon occurring in many engineering processes. For retrograde fluids, i.e. fluids with high specific heat capacities, such flows can be approximated by assuming the temperature is a constant. The one-dimensional case of the system describing such flows in Lagrangian coordinates is

(1.1) (1.1)
$$\begin{aligned} &v_t - u_x = 0,\\ &u_t + p(\lambda, v)_x = \epsilon u_{xx},\\ &\lambda_t = \frac{1}{\gamma} w(\lambda, v) + \beta \lambda_{xx}, \end{aligned} \tag{1.1}$$

where v is the specific volume, u the velocity of the fluid, λ the weight portion of vapor in the liquid/vapor mixture, ϵ the viscosity, β the

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