

**Cooperative Distributed Scheduling for Storage Devices in Microgrids using Dynamic
KKT Multipliers and Consensus Networks**

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Abstract—Scheduling of storage devices in microgrids with multiple renewable energy resources is crucial for their optimal and reliable operation. With proper scheduling, the storage devices can capture the energy when the renewable generation is high and utility energy price is low, and release it when the demand is high or utility energy price is expensive. This scheduling is a multi-step optimization problem where different time-steps are dependent on each other. Conventionally, this problem is solved centrally. The central controller should have access to the real-time states of the system as well as the predicted load and renewable generation information. It should also have the capability to send dispatch commands to each storage device. However, as the number of devices increases, the centralized approach would not be scalable and will be vulnerable to single point of failure. Combining the idea of dynamic KKT multipliers with consensus networks, this paper introduces a novel algorithm that can optimally schedule the storage devices in a microgrid solely through peer-to-peer coordination of devices with their neighbors without using a central controller.

Index Terms— Microgrids; Optimal Scheduling; Distributed Algorithms; Optimal Control;

I. INTRODUCTION

Microgrids are small scale power systems with local generation resources, storage devices [1] and loads. They are the vital elements to integrate distributed renewable energy resources to the grid [2]. The storage devices in the microgrids can capture the electric energy of the renewables when generation is more than demands or the utility electricity price is low, and then release it during high demands or when the electricity is expensive.

In order to operate microgrids efficiently, the energy resources and storage devices need to be optimally scheduled. This scheduling has to take real-time operating conditions of the system as well as the predicted information regarding the future renewable generations and demands into consideration [3]. Because of the storage devices, the amount of energy generated/consumed in one time step is not independent of other time steps. Therefore, the scheduling becomes a multi-step decision making process and typically centralized technologies are proposed to solve it. For example, centralized cooperative particle swarm algorithm in [4], centralized mixed integer linear programming in [5], and genetic algorithms in [6] are used for optimal allocation of resources in microgrids.

As the number of devices in microgrids increases and microgrids are further combined to form smart grids, the

centralized solutions would not be efficient anymore [7]. First of all, they are required to have communications with all the controllable devices in the system, which will increase the congestion. Second, they are not scalable with the increasing number of devices. And third, they are vulnerable to single points of failure. For this reason, in recent years, in the literature of smart grids, distributed solutions have attracted attention. For instance, distributed load shedding [8], distributed economic dispatch for energy resources [9]–[11], distributed generation and demand response [7], and distributed electric vehicle charge optimization [12]. However, many of the proposed distributed approaches only solve single step optimization problems. In [13], [14], and [15] multi-step distributed optimizations are introduced based on Lagrange multipliers to solve the microgrid scheduling problem, but the distributed controllers still need to communicate with all the other controllers regarding their common binding constraints.

This paper provides a novel cooperative distributed algorithm for optimal scheduling of storage devices in a microgrid. The algorithm combines the concepts of dynamic KKT multipliers and consensus networks. In this approach, each energy device in the microgrid is equipped with a distributed controller. The distributed controllers have the ability to exchange information with their neighbors. Provided that the communications network among distributed controllers is connected, each distributed controller can find the optimal schedule for its connected device in an iterative procedure. Three main features of the proposed algorithm are as follows:

- 1) Solves the multi-step optimization problem with temporally and spatially coupled constraints in a fully distributed way without requiring any central controller/coordinator/leader.
- 2) The nodes are only required to communicate with their neighbors. Unlike some distributed scheduling approaches in the literature, the devices do not have to communicate with all the devices that share the same binding constraints.
- 3) Devices do not have to disclose their consumption/generation information to other devices. The information being exchanged is only the estimations of global primal/dual variables.

Organization of the paper is as follows. Section II formulates the optimal scheduling of the storage devices in the microgrid as a discrete optimal control problem. Section III explains the proposed distributed scheduling algorithm. Section IV

demonstrates the performance of the algorithm on the Future Renewable Electric Energy Delivery and Management (FREEDM) System [16] as a realization of a future smart grid. Finally, section V summarizes the paper and brings the concluding remarks.

II. PROBLEM FORMULATION

The microgrid of interest in this paper consists of distributed renewable resources, storage devices and local loads. Scheduling of storage devices in this type of microgrid can be formulated as a discrete-time optimal control problem. To this end, we define the controllable/uncontrollable inputs, states, objective function, and constraints. In the remainder of the paper, it is assumed that all the devices in the microgrid are assigned with a unique index, and the set of indices of all devices is denoted by I .

A. Controllable Inputs

The vector of controllable inputs is denoted by $\mathbf{u}(t)$, which consists of:

- 1) Power commands to storage devices: The power command to the storage device with index i at time step t is denoted by $P_{i,B}(t)$, where $i \in B$, and B is the set of indices of storage units. A positive value denotes injecting power into the microgrid.
- 2) Power command to the grid interface: The power to be drawn from the grid at time step t is denoted by $P_{i,grid}(t)$ where $i \in grid$ is the index assigned to the grid interface, and the set containing this index is denoted by $grid$.

B. Uncontrollable Inputs

The vector of uncontrollable inputs is denoted by $\mathbf{w}(t)$ which consists of:

- 1) Renewable generation: The power produced by the renewable generation unit with index i at time step t is denoted by $P_{i,R}(t)$, where $i \in R$, and R is the set of indices of renewable units.
- 2) Demands: The power consumed by the load unit with index i at time step t is denoted by $P_{i,D}(t)$, where $i \in D$, and D is the set of indices of demand units.
- 3) Energy price: The price of energy at the outer grid at time step t is denoted by $p(t)$.

C. System States

Microgrid states are the values of stored energy in storage devices. The energy stored at energy storage device with index i at time t is denoted by $x_i(t)$, which has the following dynamics:

$$\forall i \in B : x_i(t+1) = x_i(t) - P_{i,B}(t)\Delta t, \quad (1)$$

where Δt is the time interval between two time steps (for instance 1 hour). The vector of states is denoted by $\mathbf{x}(t) = [x_i(t) : i \in B]$.

D. Objective Function

The objective is to minimize the operational cost of the microgrid over a specified time horizon,

$$\min_{\mathbf{u}(t), t=1, \dots, T} \left(J = \sum_{t=1}^T \gamma^{t-1} C(\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)) \right), \quad (2)$$

where $C(\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t))$ is the operational cost of the microgrid at time-step t . Here, $0 < \gamma \leq 1$ is a discount factor to put less weight on future values of the costs. In this paper, we are not considering any thermal plants installed in the microgrid, so the only operational cost is due to the energy bought from the utility. Therefore,

$$C(\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)) = p(t)P_{i,grid}(t)\Delta t. \quad (3)$$

E. Constraints

Three classes of constraints should be satisfied:

- 1) *Power Balance Constraint*: At all times, the amount of generation should equal to the amount of load:

$$\forall 1 \leq t \leq T : \sum_{i \in B} P_{i,B}(t) + \sum_{i \in R} P_{i,R}(t) + P_{i,grid}(t) = \sum_{i \in D} P_{i,D}(t). \quad (4)$$

Note that in this paper, we do not consider losses.

- 2) *Energy Constraint for Storage Devices*: At all times, the energy stored in the storage devices cannot be more than their capacity or negative:

$$\forall 1 \leq t \leq T, i \in B : 0 \leq x_i(t) \leq E_{i,full}, \quad (5)$$

where $E_{i,full}$ is the total amount of energy that can be stored in the storage device with index i . Combining (1) and (5):

$$\forall 1 \leq s \leq T, i \in B : x_{i0} - E_{i,full} \leq \sum_{t=1}^s P_{i,B}(t)\Delta t \leq x_{i0}, \quad (6)$$

where x_{i0} is the initial value of the stored energy at the storage device with index i .

- 3) *Power Rating Constraints*: At all times, the power commands sent to the storage devices or to the grid interface should be within the maximum and minimum limits:

$$\forall 1 \leq t \leq T : \begin{cases} P_{i,B \min} \leq P_{i,B}(t) \leq P_{i,B \max} \\ P_{grid \min} \leq P_{i,grid}(t) \leq P_{grid \max} \end{cases}, \quad (7)$$

where $P_{i,B \min}$, $P_{i,B \max}$ are the minimum and maximum power limits of the storage device with index i , and $P_{grid \min}$, $P_{grid \max}$ are the minimum and maximum power that can be drawn from the grid.

III. DISTRIBUTED PREDICTIVE MICROGRID SCHEDULING ALGORITHM

In this section, the steps to derive the algorithm to solve the formulated optimal control problem in a distributed approach are explained.

A. Lagrangian of the problem

The augmented Lagrangian of the problem by adding KKT multipliers, constraints, and penalty terms to the objective function would be:

$$\begin{aligned} J = & \sum_{t=1}^T \gamma^{t-1} p(t)P_{i,grid}(t) \\ & + \sum_{t=1}^T \lambda(t) \left(\sum_{i \in D} P_{i,D}(t) - \sum_{i \in B} P_{i,B}(t) - \sum_{i \in R} P_{i,R}(t) - P_{i,grid}(t) \right) \\ & + \sum_{t=1}^T \sum_{i \in B} \mu_i(t) \left(x_{i0} - E_{i,full} - \sum_{s=1}^t P_{i,B}(s)\Delta t \right) \\ & + \sum_{t=1}^T \sum_{i \in B} \xi_i(t) \left(\sum_{s=1}^t P_{i,B}(s)\Delta t - x_{i0} \right) \end{aligned} \quad (8)$$

$$\begin{aligned}
& + \frac{\rho}{2} \sum_{t=1}^T \left(\sum_{i \in D} P_{i,D}(t) - \sum_{i \in B} P_{i,B}(t) - \sum_{i \in R} P_{i,R}(t) - P_{i,grid}(t) \right)^2 \\
& + \frac{\rho}{2} \sum_{t=1}^T \sum_{i \in B} \left[\left[x_{i0} - E_{i,full} - \sum_{s=1}^t P_{i,B}(s) \Delta t \right]^+ \right]^2 \\
& + \frac{\rho}{2} \sum_{t=1}^T \sum_{i \in B} \left[\left[\sum_{s=1}^t P_{i,B}(s) \Delta t - x_{i0} \right]^+ \right]^2,
\end{aligned}$$

where $\lambda(t)$, $\mu_i(t)$, and $\zeta_i(t)$ are the KKT multipliers for the constraints, and $[\]^+$ projects its argument to the positive values. The last three terms are penalty terms for the constraints and improve the convergence properties of the iterative algorithms [17], and ρ is the penalty factor.

B. Update Equations based on Gradient Descent/Ascent

The solution to the optimal control problem can be found in an iterative procedure by moving the primal variables (controllable inputs) in the opposite direction and dual variables (KKT multipliers) in the direction of the gradient of the Lagrangian [18]. Therefore, the update equations would be:

$$\forall t \in \{1, \dots, T\}: \quad P_{i,grid}^{k+1}(t) = P_{i,grid}^k(t) - \eta \left(\gamma^{k-1} p(t) - \lambda^k(t) - \rho \Delta P^k(t) \right), \quad (9)$$

$$\forall t \in \{1, \dots, T\}, i \in B:$$

$$P_{i,B}^{k+1}(t) = P_{i,B}^k(t) - \eta \begin{pmatrix} -\lambda^k(t) - \rho \Delta P^k(t) \\ -\sum_{l=t}^T \mu_i^k(l) \Delta t + \sum_{l=t}^T \zeta_i^k(l) \Delta t \\ -\rho \Delta t \sum_{l=t}^T \left[x_{i0} - E_{i,full} - \sum_{s=1}^l P_{i,B}^k(s) \Delta t \right]^+ \\ + \rho \Delta t \sum_{l=t}^T \left[\sum_{s=1}^l P_{i,B}^k(s) \Delta t - x_{i0} \right]^+ \end{pmatrix}, \quad (10)$$

$$\forall t \in \{1, \dots, T\}: \quad \lambda^{k+1}(t) = \lambda^k(t) + \eta \Delta P^k(t), \quad (11)$$

$$\forall t \in \{1, \dots, T\}, i \in B:$$

$$\mu_i^{k+1}(t) = \left[\mu_i^k(t) + \eta \left(x_{i0} - E_{i,full} - \sum_{s=1}^t P_{i,B}(s) \Delta t \right) \right]^+, \quad (12)$$

$$\forall t \in \{1, \dots, T\}, i \in B:$$

$$\zeta_i^{k+1}(t) = \left[\zeta_i^k(t) + \eta \left(\sum_{s=1}^t P_{i,B}^k(s) \Delta t - x_{i0} \right) \right]^+, \quad (13)$$

where,

$$\forall t \in \{1, \dots, T\}: \quad \Delta P^k(t) = \sum_{i \in D} P_{i,D}^k(t) - \sum_{i \in B} P_{i,B}^k(t) - \sum_{i \in R} P_{i,R}^k(t) - P_{i,grid}^k(t). \quad (14)$$

By choosing small enough positive values for η and ρ , the update equations (9)-(13) would converge to the saddle point of the Lagrangian which is the optimal point of the problem [18]. However, using equations (9)-(11) requires each node having access to certain global information of the microgrid which are the forecasted load of all the demand units, forecasted generation of all the renewable generation units, and controllable inputs decided for all the dispatchable nodes.

C. Consensus Networks to Estimate Global Information

To make the algorithm presented by (9)-(13) fully distributed, instead of using global information, we use local estimations of global information at each node and allow the nodes to coordinate their estimations with their neighbors through consensus networks [19]. Thus, equations (9) and (10) become:

$$\forall t \in \{1, \dots, T\}: \quad P_{i,grid}^{k+1}(t) = P_{i,grid}^k(t) - \eta \left(\gamma^{k-1} p(t) - \hat{\lambda}_i^k(t) - \rho \Delta \hat{P}_i^k(t) \right), \quad (15)$$

$$\forall t \in \{1, \dots, T\}, i \in B:$$

$$P_{i,B}^{k+1}(t) = P_{i,B}^k(t) - \eta \begin{pmatrix} -\hat{\lambda}_i^k(t) \\ -\rho \Delta \hat{P}_i^k(t) - \sum_{l=t}^T \mu_i^k(l) \Delta t + \sum_{l=t}^T \zeta_i^k(l) \Delta t \\ -\rho \Delta t \sum_{l=t}^T \left[x_{i0} - E_{i,full} - \sum_{s=1}^l P_{i,B}^k(s) \Delta t \right]^+ \\ + \rho \Delta t \sum_{l=t}^T \left[\sum_{s=1}^l P_{i,B}^k(s) \Delta t - x_{i0} \right]^+ \end{pmatrix}, \quad (16)$$

where $\Delta \hat{P}_i^k(t)$ and $\hat{\lambda}_i^k(t)$ are the estimations of node i at iteration k from the average of the power imbalance in the network, $\Delta P^k(t)$, and the dual variable, $\lambda^k(t)$, respectively.

The estimation variables are updated in all the nodes as follows:

$$\forall t \in \{1, \dots, T\}, i \in B \cup D \cup R \cup grid, j \in \{B, D, R, grid\}:$$

$$\hat{\lambda}_i^{k+1}(t) = \hat{\lambda}_i^k(t) + \sum_{j \in N_i} w_{ij} \left(\hat{\lambda}_j^k(t) - \hat{\lambda}_i^k(t) \right) + \eta \Delta \hat{P}_i^k(t), \quad (17)$$

$$\Delta \hat{P}_i^{k+1}(t) = \Delta \hat{P}_i^k(t) + \sum_{j \in N_i} w_{ij} \left(\Delta \hat{P}_j^k(t) - \Delta \hat{P}_i^k(t) \right) + P_{i,j}^k(t) - P_{i,j}^{k+1}(t),$$

where N_i is the set of neighbors of node i in the communications network and $w_{ij} = w_{ji}$ is the connectivity strength between node i and node j and is chosen such that $0 < w_{ij} < \left(\max_{i=1, \dots, N} |N_i| \right)^{-1}$ to satisfy the convergence criteria for the consensus network [19].

Devices such as transformers/converters in the microgrid do not actively generate/consume power. However, from the communications point of view, they might participate in the information exchange process. Therefore, for all those devices, the estimation variables are updated as:

$$\forall t \in \{1, \dots, T\}, \forall i \in I \setminus (B \cup R \cup D \cup grid):$$

$$\hat{\lambda}_i^{k+1}(t) = \hat{\lambda}_i^k(t) + \sum_{j \in N_i} w_{ij} \left(\hat{\lambda}_j^k(t) - \hat{\lambda}_i^k(t) \right) + \eta \Delta \hat{P}_i^k(t), \quad (18)$$

$$\Delta \hat{P}_i^{k+1}(t) = \Delta \hat{P}_i^k(t) + \sum_{j \in N_i} w_{ij} \left(\Delta \hat{P}_j^k(t) - \Delta \hat{P}_i^k(t) \right).$$

IV. NUMERICAL SIMULATIONS

To validate the proposed approach, we consider application of the algorithm on a microgrid configuration shown in Fig 1. The figure shows a typical configuration of the FREEDM System which is a realization of the smart microgrids [16]. It consists of three Distributed Energy Storage Devices (DESDs), a wind turbine, a PV panel, two load nodes as well as Solid State Transformers (SSTs) which are power electronics converters. On top of the physical system, there is the communications network through which different devices can exchange information. The nodes are shown with circles and the communication links are shown with dotted lines. The existence

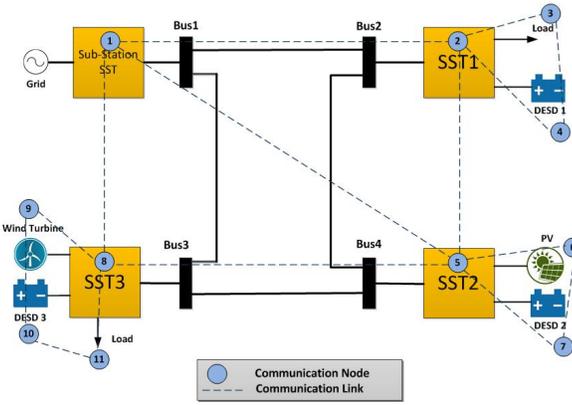


Fig 1. Typical configuration of the FREEDM system

of a communication link between two nodes denotes that they are neighbors. All the simulations are performed in MATLAB 2013a, on a PC with 2.50 GHz processor and 8 GB RAM.

The local information regarding power generation and consumption of each device is only accessible to the controller connected to that device. For instance, only the distributed controller located at node 1 has access to the load forecast information of the load connected to node 1. Similarly only node 6 has access to the generation forecast data of the PV panel connected to that node.

The objective is that the devices collaboratively coordinate with each other and find the optimal schedule for the next 5 hours. As the case study, we have used online data for price profile¹, and scaled profiles for PV², wind² and loads³ as shown in Table 1. The information regarding the specifications of DESDs, and their initial stored energy is shown in Table 2.

The proposed algorithm is applied. The value of $\eta = \rho$ is chosen as 0.35, the discount factor is chosen as $\gamma = 1$, and all the connectivity strengths are chosen as $w_{ij} = 0.17$ to satisfy $0 < w_{ij} < \left(\max_{i=1,\dots,N} |N_i|\right)^{-1} = 1/5$.

Hour	Price (cents/kWh)	Wind (kW)	Solar (kW)	Load1 (kW)	Load2 (kW)
14:00	4.33	1.8	3.8	4.3	8.8
15:00	4.24	1.9	2.5	4.3	8.7
16:00	4.22	2.1	1.3	4.2	8.7
17:00	5.76	2.1	0.4	4.4	8.7
18:00	8.34	2.2	0.0	4.8	9.4

	Cap (kWh)	$P_{i,Bmax}$ (kW)	x_{i0} (kWh)
DESD 1	5	5	1
DESD 2	10	5	1.5
DESD 3	5	5	0.5

Fig 2 shows the evolution of the estimations of the dual variables $\lambda_i(t)$. Fig 2(a) shows the evolution of dual variable estimations for different time steps at node 1. Fig 2(b) - Fig 2(f) show the evolution of dual variable estimations for each time step by all other nodes. It can be seen that the estimations of different nodes of the system for the dual variable of each time step converge to the same value. However the dual variables of different time steps (as shown in Fig 2 (a)) do not have to be equal as they correspond to different constraints governing each

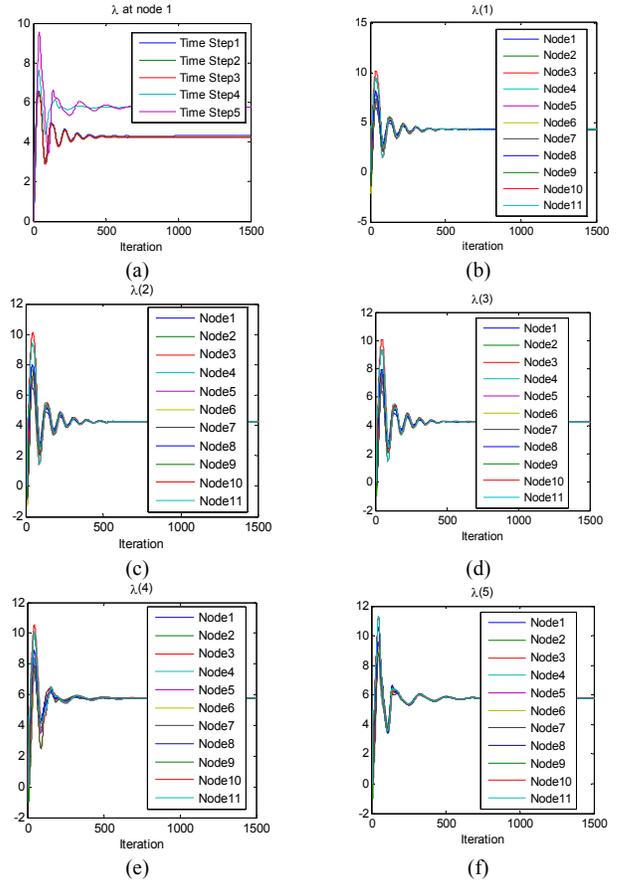


Fig 2. Evolution of estimations of dual variable λ

time step. The resultant schedule for the DESDs along with the load, wind and PV generation profiles are shown in Fig 3.

It can be seen that during the time steps when the price is low (time steps 2, and 3) and the renewable generation is relatively high, the DESDs are charging, and when the price goes high and renewable generation decreases, the DESDs discharge to support the demand, and avoid high operational costs.

To demonstrate the optimality of the resultant schedule, we found the global optimum using centralized Linear Programming (LP). Table 3 shows the objective value and convergence time of the proposed algorithm benchmarked against LP. As the proposed algorithm is an iterative process, the convergence time depends on the stopping criteria. The summation of L2 norms of all constraint violations as well as the relative change in the decision variables is defined as the Convergence Index (CI). The algorithm is stopped once $CI < \epsilon$. Table 3 shows the convergence time and the resultant objective value for different values of ϵ . We can see that the smaller the value of ϵ is, the closer the solution is to the global optimum, but it takes longer time. For this small scale problem, as the centralized approach has access to the entire information, it can find the solution faster than the distributed algorithm in which each node has only partial information. To see how the scale of the problem affects the computational complexity of the algorithm, we increased the number of devices from 10 to 105 and for each case performed 30 random scenarios. Fig 4 shows the median of the iterations required for convergence ($CI <$

¹ <http://www.iso-ne.com/markets-operations/markets/da-rt-energy-markets>

² <http://www.transparency.eex.com/en/>

³ <http://pjm.com/markets-and-operations/energy/real-time/hourly-prelim-loads.aspx>

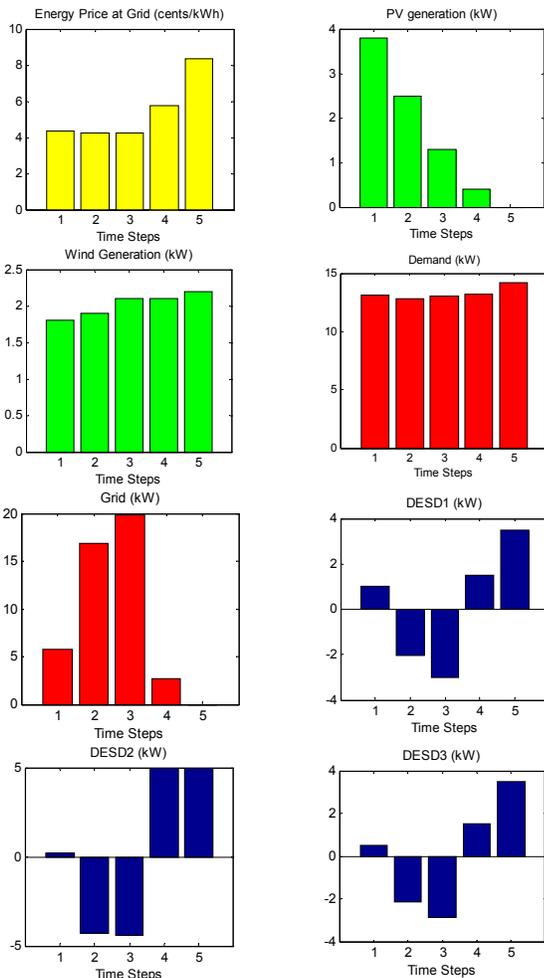


Fig 3. Forecasted profiles and resultant schedule using the proposed algorithm

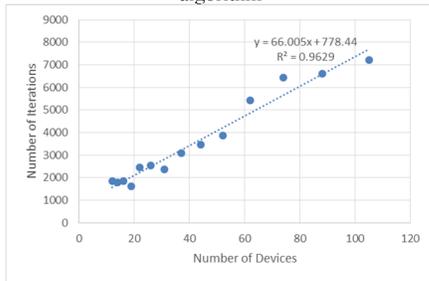


Fig 4. Median of convergence iterations vs. number of devices for $\epsilon = 0.01$

TABLE 3. BENCHMARK AGAINST CENTRALIZED ALGORITHM		
Algorithm	Objective Value (cents)	Convergence Time (sec)
LP	195.9938	0.4768
Proposed Algorithm ($\epsilon=1e-1$)	197.2430	0.5248
Proposed Algorithm ($\epsilon=1e-2$)	195.9910	2.1477
Proposed Algorithm ($\epsilon=1e-3$)	195.9870	2.7509
Proposed Algorithm ($\epsilon=1e-6$)	195.9938	3.9829

0.01) as a function of number of devices which indicates a linear increase in the computational complexity.

V. CONCLUSION

This paper introduced a distributed predictive algorithm for optimal scheduling of microgrids consisting of renewable generations and storage devices. Considering more realistic

assumptions such as storage device efficiency, degradation, losses, etc. would be the future work of the authors.

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